College of Education
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Developing Early Number Sense for the Struggling Learners

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POWERTHEKNOWLEDGE

Numerical Magnitude

- Recognizing that a number of quantity is more than, less than, or equal to another number of quantity (cf. Gersten et al., 2012)
- Critical for number sense & number competence (Booth & Siegler, 2008; Gersten et al., 2012)
- Performance in numerical magnitude, highly predictive and/or related to mathematics achievement (De Smedt, Noel, Gilmore & Ansair, 2013; Fazio et al., 2014).

Research Study: Overview

- Data comes from a larger study focused on how students develop ideas in number and operation and the cognitive difficulties they experience.
- Longitudinal + teaching experiment design
- Two public schools in mid-western United States.
- 78 students
  - 1st grade students (year 1), 2nd grade students (year 2)
- Data collected at 4 points over the 2-years
- Early Number Battery (ENB)
  - Researcher designed
  - Critical constructs within number and operation including numerical magnitude

Development in Numerical Magnitude

- Jordan and Levine (2009)
  - Six or Seven Years:
    Discriminate oral or symbolic quantities

- Sarama and Clements (2009)
  - Age Five: Counting to Compare Numbers

Benchmarking to Compare Numbers

  - Age Seven: Place Value to Compare Numbers

What are benchmarks?

- Numerical Benchmarks
  - "Essential mental referents for thinking about numbers" (McIntosh et al., 1993, p. 6).

  - Examples:
    - Multiples of 10
    - Powers of 10
    - ½, ¼, ¾
    - Values with which the learner is confident

Benchmarking in the Curriculum

- Grade 4: Number and Operations-Fractions
  - **Extend understanding of fraction equivalence and ordering.**
    - Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. CCSS.Math.Content.4.NF.A.2
- Grade 5: Number and Operations-Fractions
  - **Use equivalent fractions as a strategy to add and subtract fractions**
    - Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.* CCSS.Math.Content.5.NF.A.2

Benchmarking in Magnitude Comparison

- **Single Benchmark Examples**
  - **T:** Which is more, 89 or 123?
  - **S:** 123 is more because it is past 100.
  - **S:** 123, because it is bigger than 100 and the other number [89] is before 100, so it [89] would be less.

Benchmarking in Magnitude Comparison

- **Multiple Benchmarks Examples**
  - **T:** Which is more, 73 or 58?
  - **S:** 58 [is smaller], it is in the fifties and the other number is in the seventies and the seventies are past the fifties.
  - **T:** Which is more, 14 or 41?
  - **S:** Because 14 is in the tens and 41 is in the forties, that [points to 41] is greater.

Difficulties Students Experience

Group One (n = 26)	Group Two (n = 7)
Counting to Compare	Counting to Compare
Benchmarking to Compare	
Robust Place Value to Compare	Incorrect or Superficial Place Value to Compare

Instructional Strategies for Developing Numerical Magnitude

- What are the important mathematical ideas that students need to understand to develop a deep understanding of numerical magnitude?
- What would/do you do to help students develop these important mathematical ideas related to numerical magnitude with your learners?

Instructional Strategies for Developing Numerical Magnitude

- Tasks need to draw out students thinking and promote movement toward more advanced thinking. Make mathematical ideas explicit.
- Sample Tasks (see Handout):
  - Physical Materials
  - Promoting Benchmarking
  - Oral and Symbolic

Thank you!

Any questions or comments?

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Magnitude Tasks

1. Magnitude Tasks (Comparing Physical Materials and Symbols)

(a) Show: 6 unifix cubes. **Say:** There are 6 blocks here.
Show: a card with the number "5" written on it. **Say:** Suppose I had this number of blocks.
Ask: Which is more?

(b) Show: 3 unifix sticks of 10 and and 4 single ones. **Ask:** How many blocks are there
Show: Number card with the number "34" written on it. **Say:** Suppose I also had this number of blocks.
Ask: Which is more?
Ask: How do you know [child's response] is more?

(c) Show: 2 unifix sticks of 10 and and 8 single ones.
Show: Number card with the number "18" written on it. **Say:** Suppose I also had this number of blocks.
Ask: Which is less?
Ask: How do you know [child's response] is less?

(d) Show: 2 unifix sticks of 10 and 3 single ones.
Show: 3 unifix sticks of 10 and 2 single ones.
Ask: Which has more blocks?
Ask: How do you know [child's response] is more?

2. Magnitude Tasks (Emphasizing the Decades)

Lay out decade number cards (0-50):

(a) Say: Without moving the cards on the table, where do you think the number (show them the number card for 24) would go? Why do you think the number goes there?
Ask: Is it closer to 20 or 30? How do you know?

(b) Say: Without moving the cards on the table, where do you think the number (show them the number card for 37) would go? Why do you think the number goes there?
Ask: Is it closer to 30 or 40? How do you know?

(c) Say: Without moving the cards on the table, where do you think the number (show them the number card for 15) would go? Why do you think the number goes there?
Ask: Is it closer to 10 or 20? How do you know?

3. Magnitude Tasks (Oral)

(a) Ask: Which is more, 16 or 25?
(b) Ask: Why is [child's response] more?
(c) Ask: Which is more, 32 or 23?
(d) Ask: Which is more, 67 or 82?
(e) Ask: Which is more, 123 or 89?
(f) Ask: Which is more, 285 or 582?
(g) Ask: Which is less, 41 or 14?
(h) Ask: Which is less, 36 or 42?

4. Magnitude Tasks (Written)

(a) Do: Place in front of the child the number "16" card and the number "25" card.
Ask: Which is more?
Ask: Why is [child's response] more?

5. Magnitude Tasks
Provide a number line with 0 on the left end and 100 on the right end.

(a) Say: "Here is a line. On one end of the line is the number 0 and on the other end is the number 100. Draw a mark to show me where you think the number 35 will go on the number line?
Ask: "Why do you think 35 goes here?"
Ask: "Is 35 closer to the zero or the 100? Why?"

(b) Say: "Draw a mark to show me where you think the number 90 will go on the number line."
Ask: "Why do you think 90 goes here?"
Ask: "Is 90 closer to the 0 or the 100? Why?"

(c) Say: "Draw a mark to show me where you think the number 50 will go on the number line."
Ask: "Why do you think 50 goes here?"
Ask: "Is 50 closer to the 35 or the 90? Why?"