Many Concepts, Few Numbers: A Novel Approach to Math Interventions

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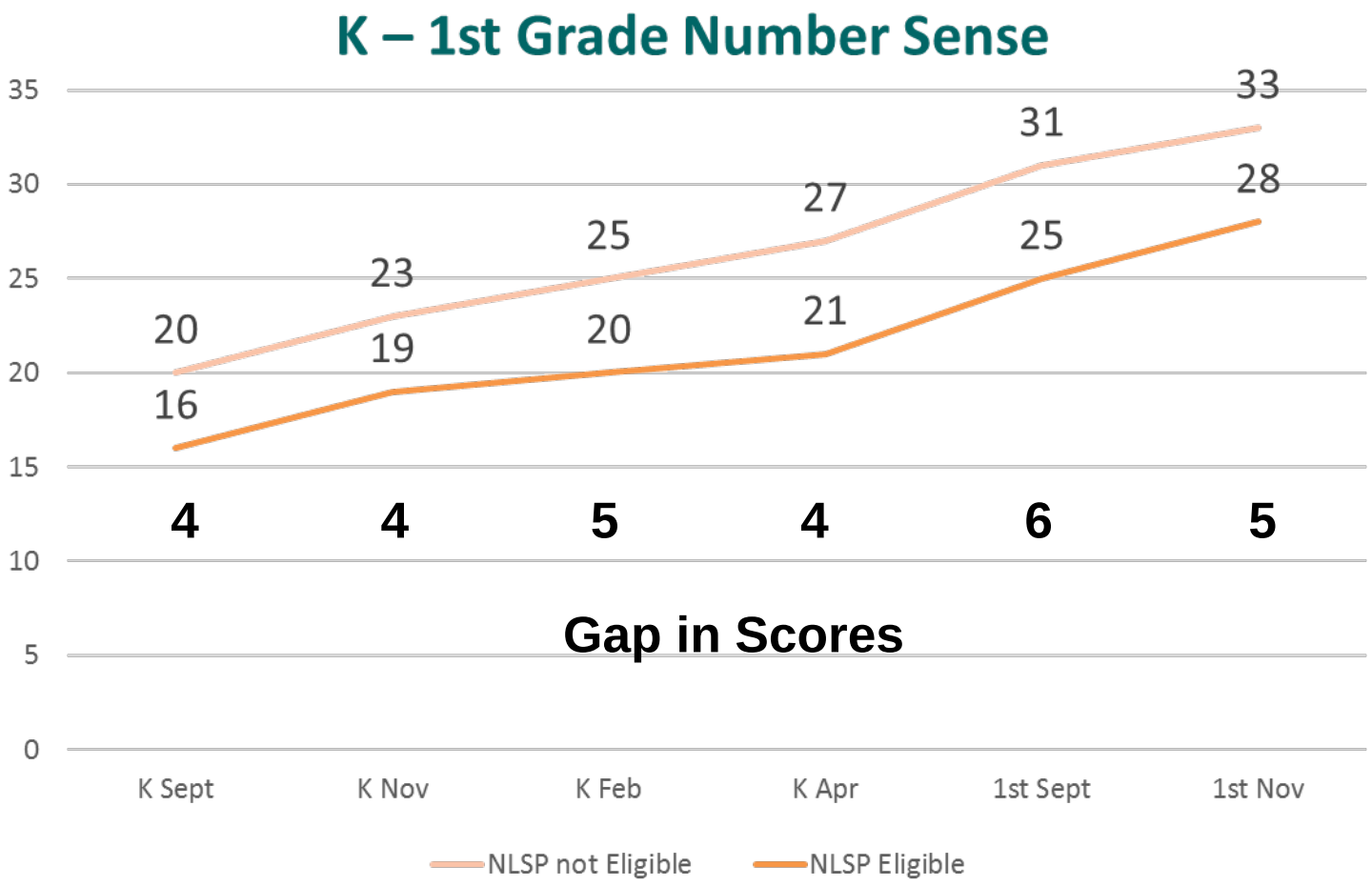
- How we arrived at this novel approach
- How we applied it to basic number sense
- How we are applying it to fraction sense
- Your Turn! Q and A and discussion around how YOU might use this approach

The Math Problem

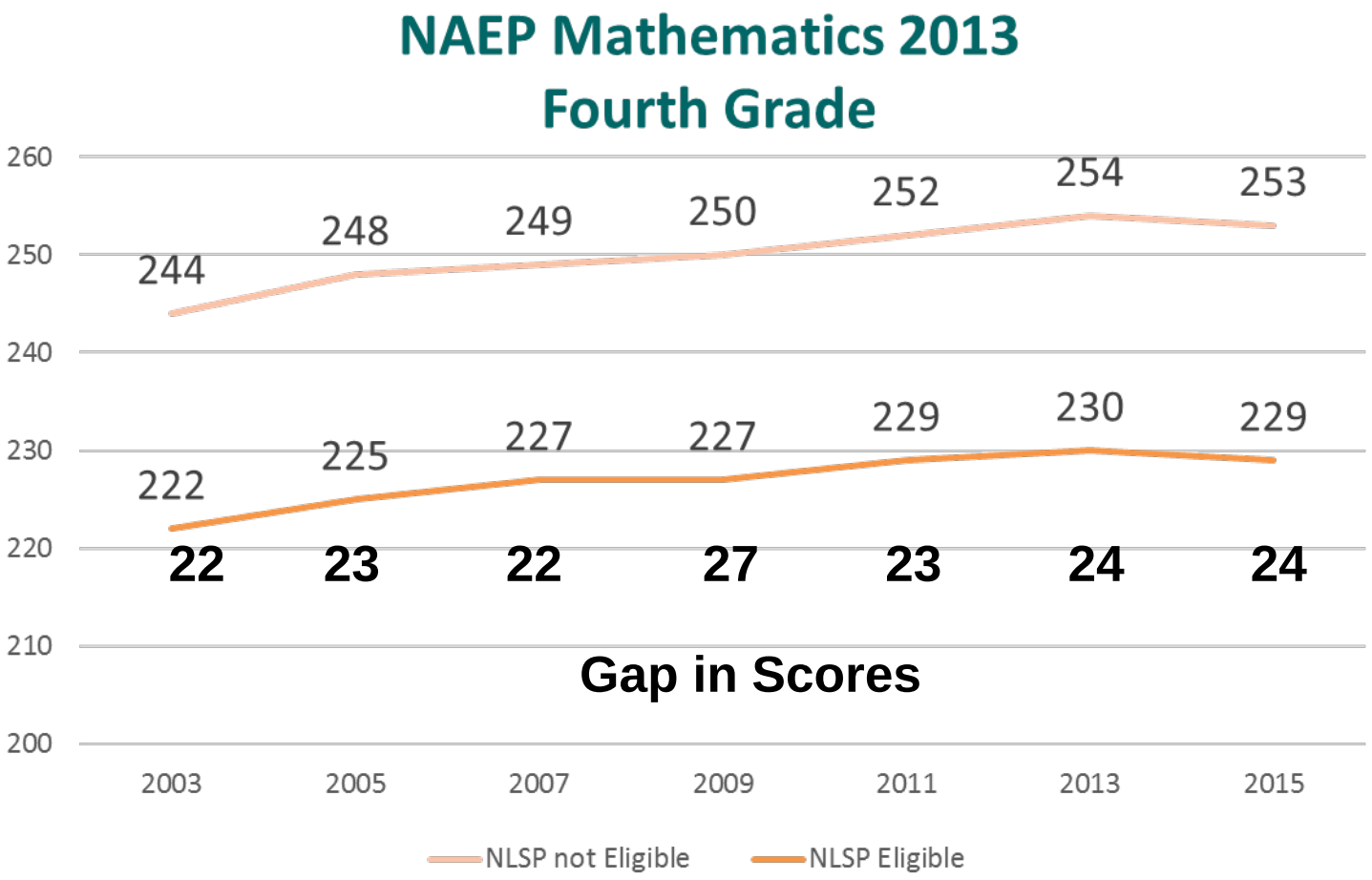
- Poor achievement in math is widespread with serious educational and vocational consequences.
- Students who do not develop foundational mathematics in elementary and middle school, are less likely to graduate from college than higher-achieving students (NMAP, 2008).
- Large math disparities exist between middle- and low-income children *before* they enter school at about five years of age (Jordan & Levine, 2009; National Research Council, 2009).

Income - Achievement Gap

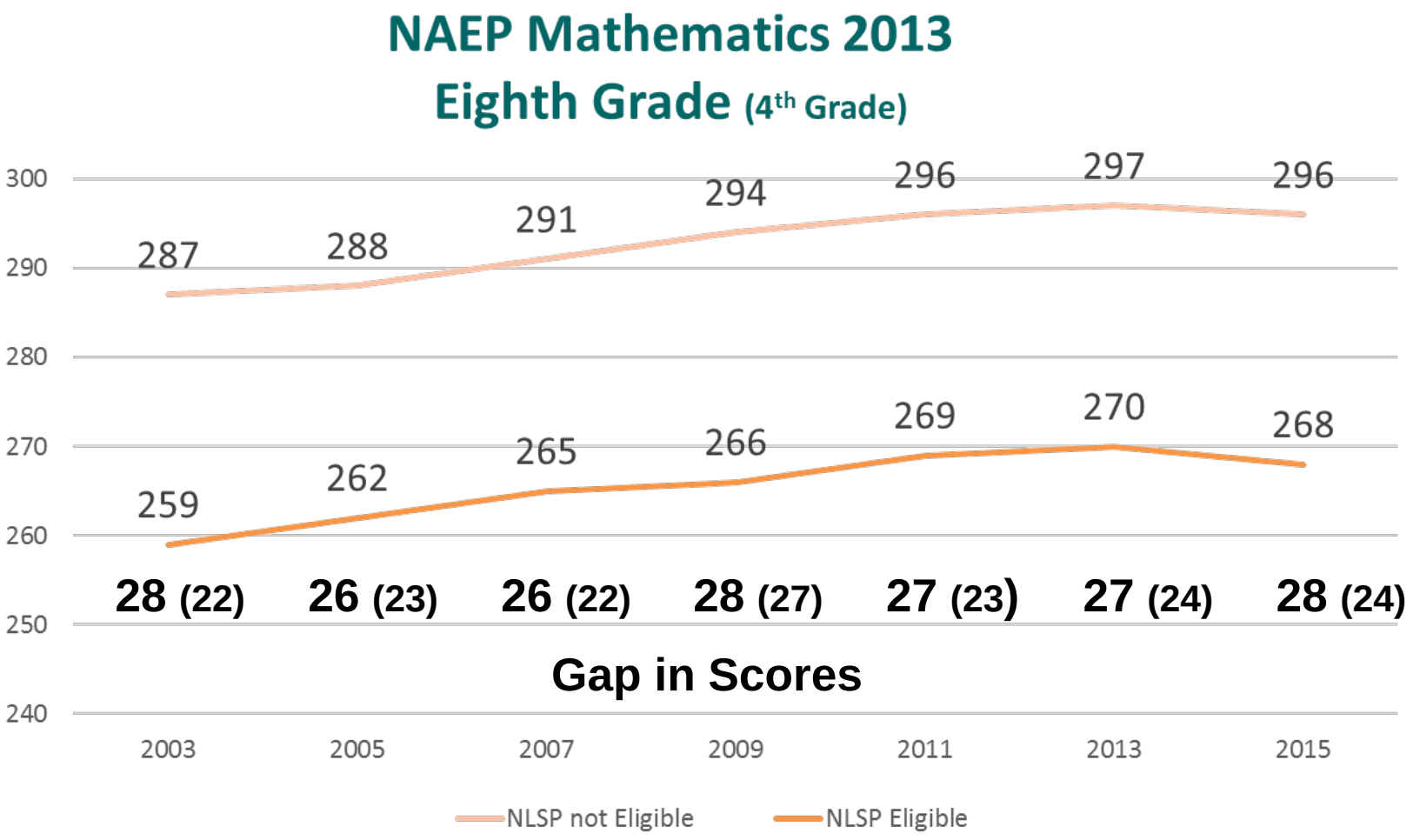
ECLS 2011



Income - Achievement Gap



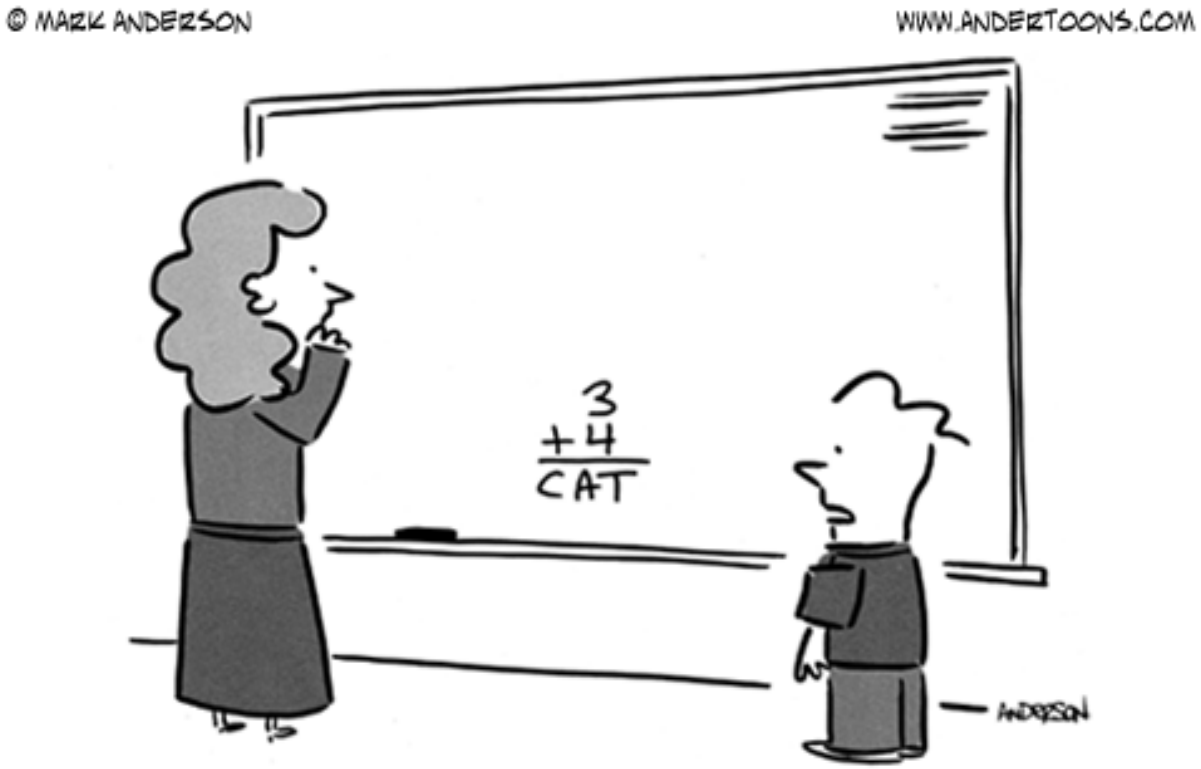
Income - Achievement Gap



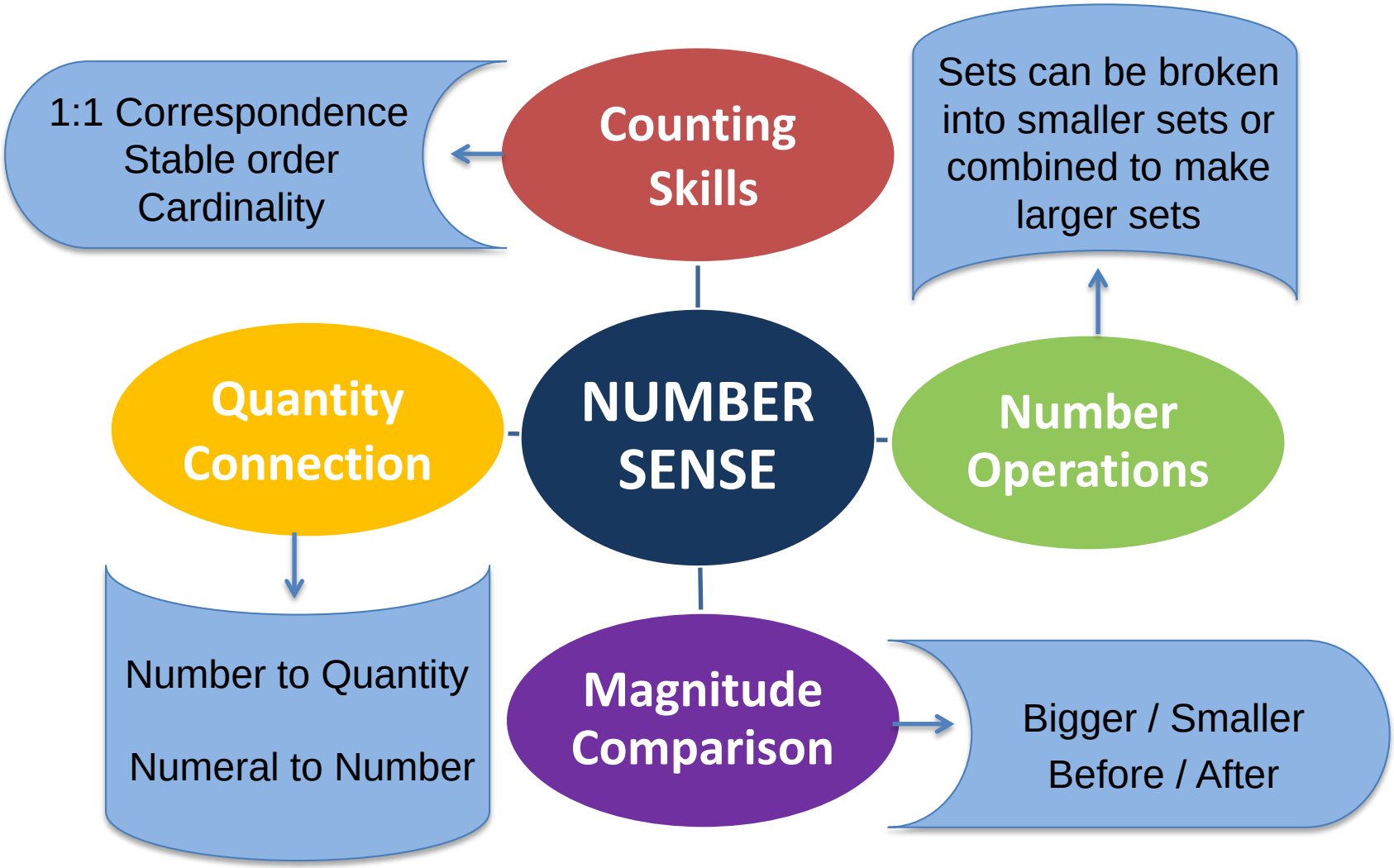
Numbers do not make sense to many children

"First they told me that 2 + 2 = 4. Then they told me that 3 + 1 = 4. That's when I lost faith in public education!"

Many times we are at a loss as to how to help them.



"On the plus side it's something for your PLC to chew on."


NUMBER SENSE
Counting Skills
1:1 Correspondence
Stable order
Cardinality
Sets can be broken into smaller sets or combined to make larger sets
Number Operations
Quantity Connection
Number to Quantity
Numeral to Number
Magnitude Comparison
Bigger / Smaller
Before / After

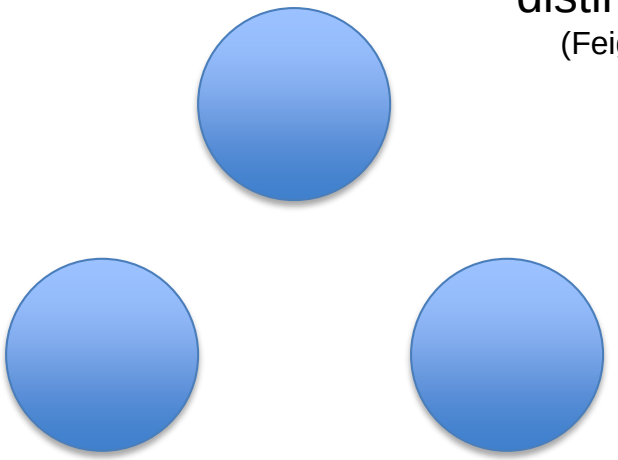
Number Sense Development

Quantity Connections

Very young children are sensitive to number
(Dehaene,1999)

Habituation studies show even 6 month old children can distinguish between quantities
(Feigenson, Dehaene, & Spelke, 2004)

***SUBITIZE* –** the ability to recognize small quantities without counting

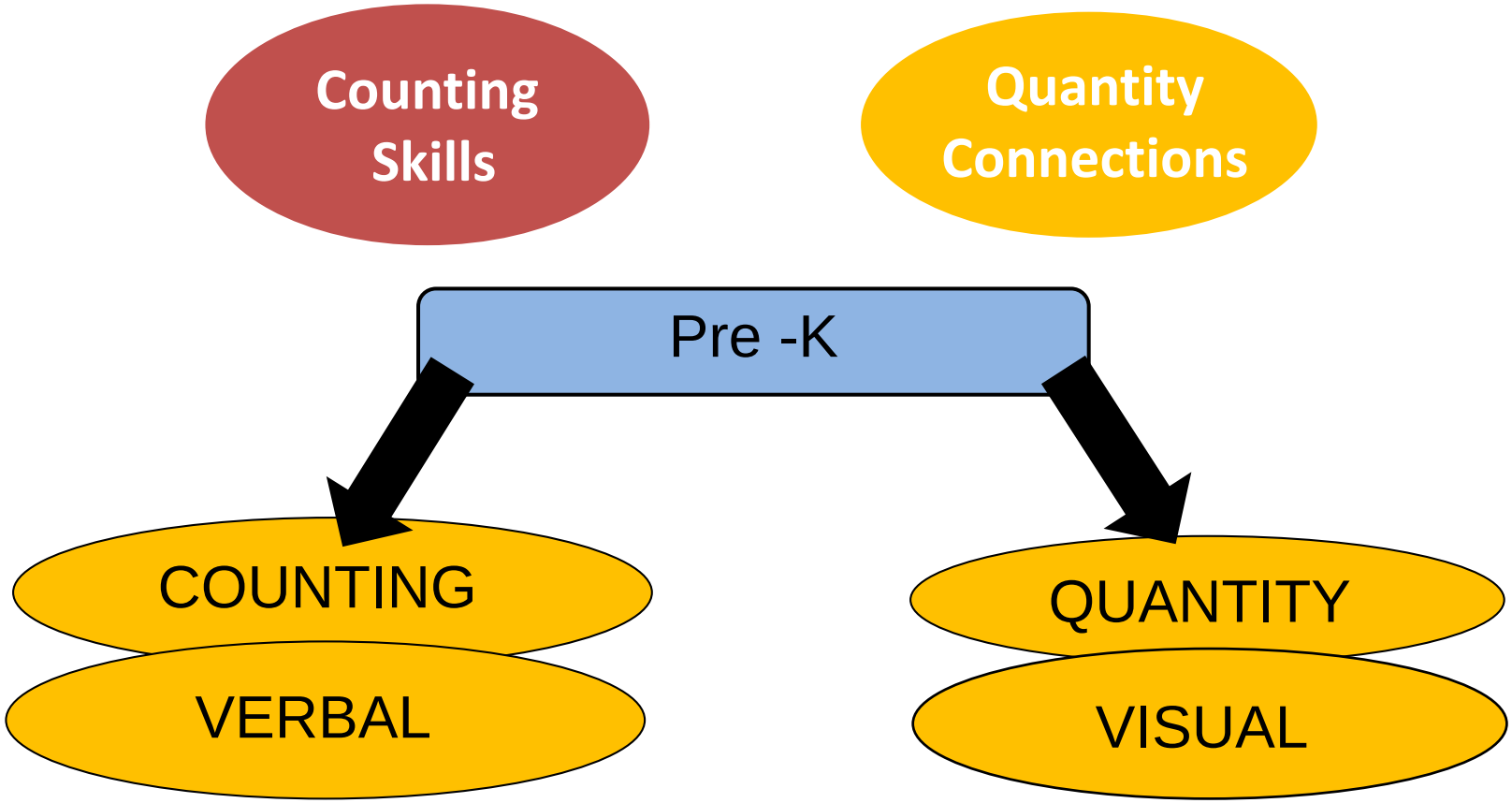


Perceptually

Conceptually

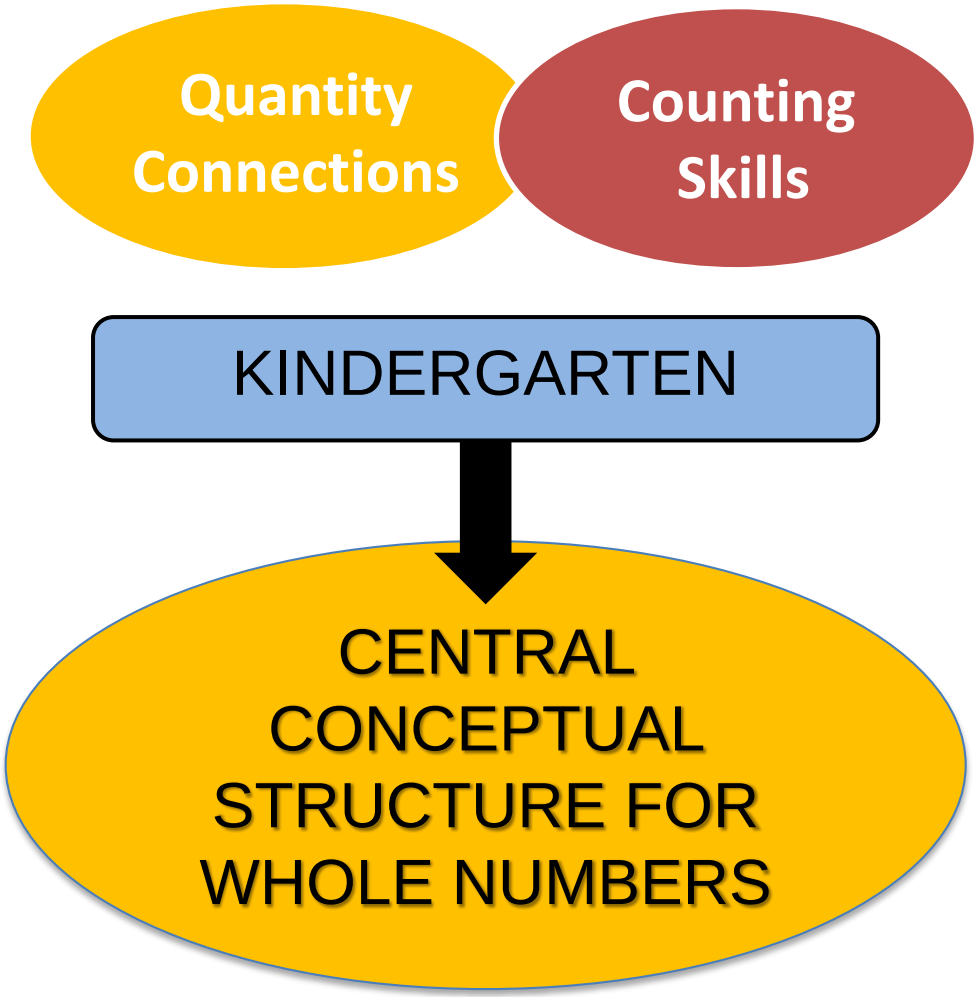
Number Sense Development

(Griffin & Case,1997)



Number Sense Development

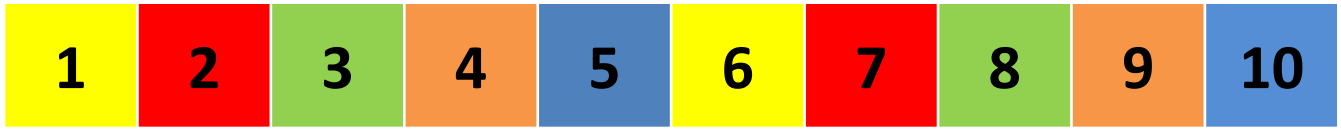
(Griffin & Case,1997)



Number Sense Development

Magnitude Comparison

MENTAL NUMBER LIST

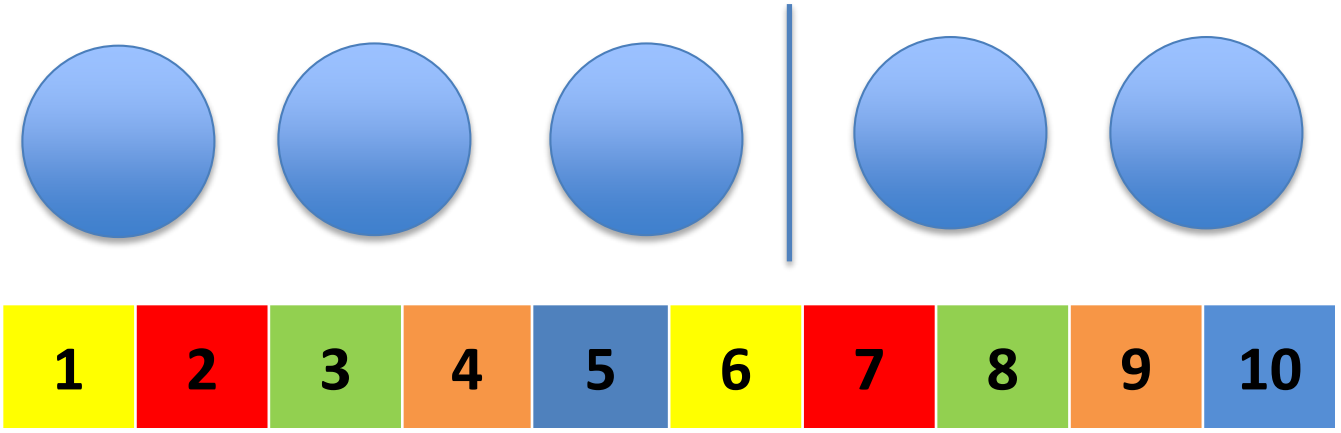


As you go up the list, the quantities get larger

Each number is one more than the number before it and one less than the number after it. (Baroody, Eiland, & Thompson, 2009)

Number Sense Development

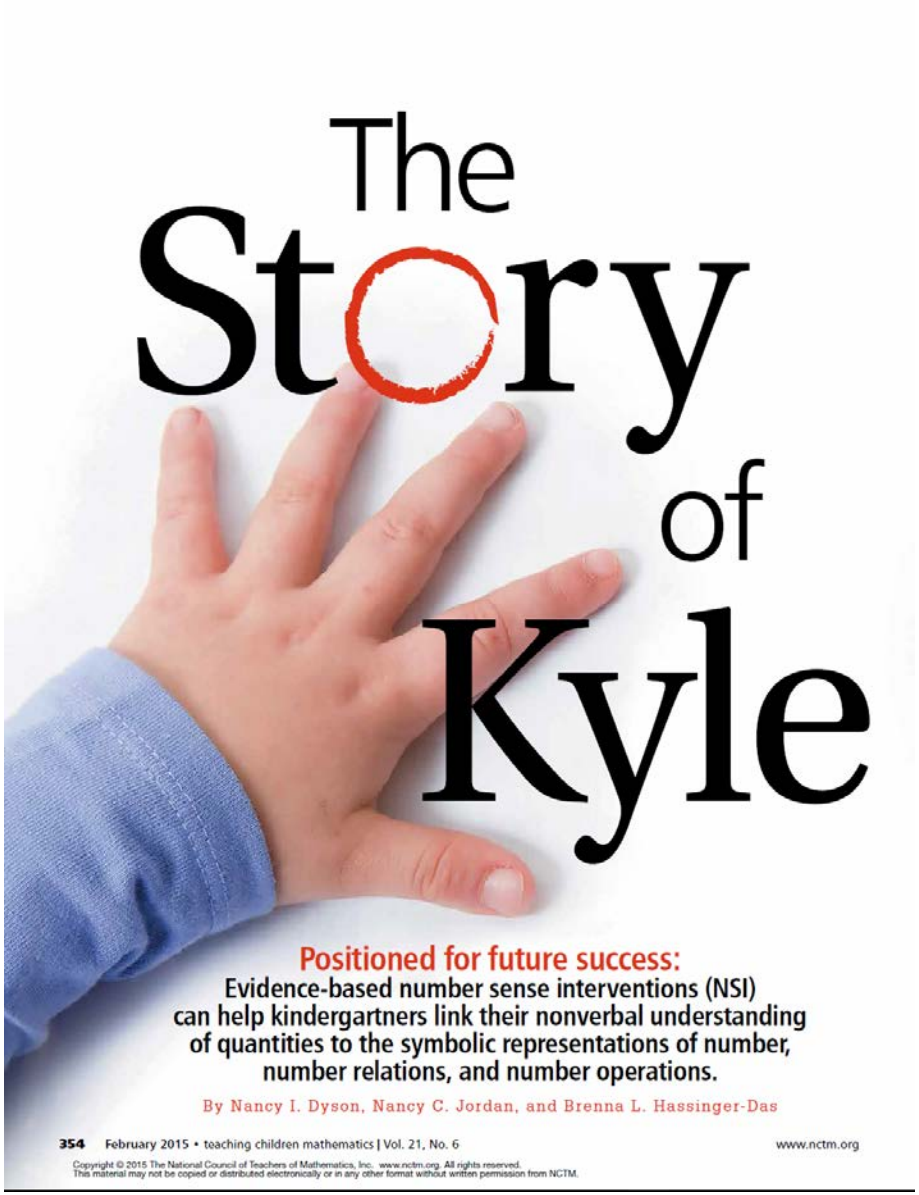
Number Operations



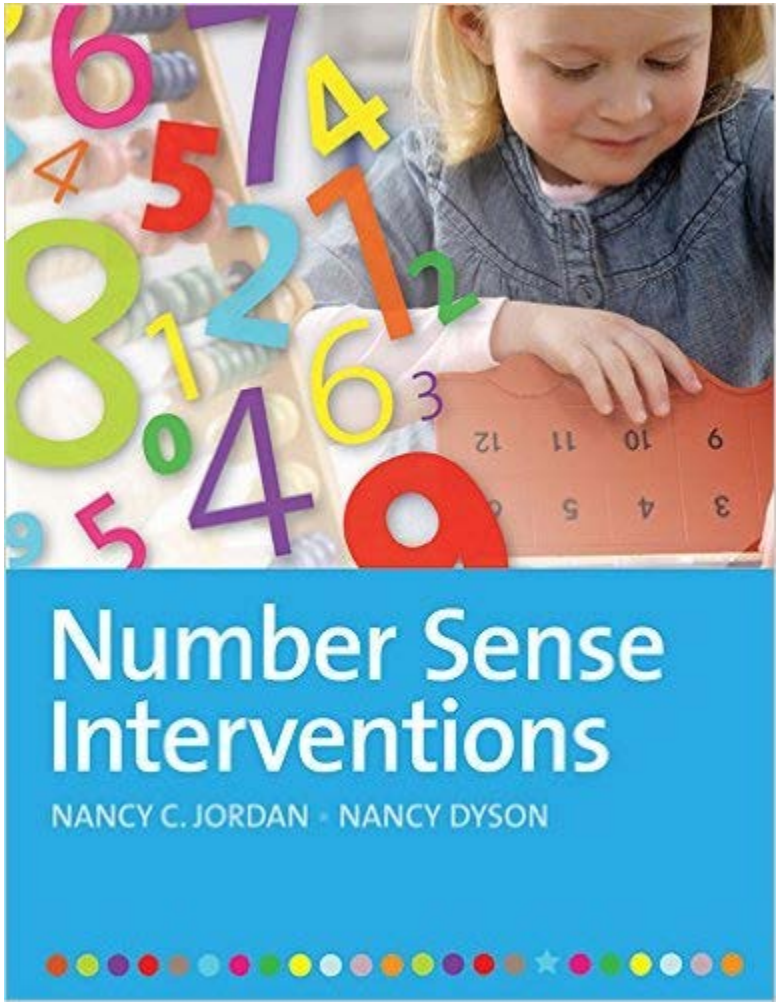
3+2=5

Developing Number Sense in Children at Risk for Mathematics Learning Disabilities

funded by the National Institute of Child Health and Human Development (5R01HD059170),
Nancy C. Jordan, Principal Investigator
2009 - 2014



TCM February 2015
Dyson, Jordan, Hassinger-Das



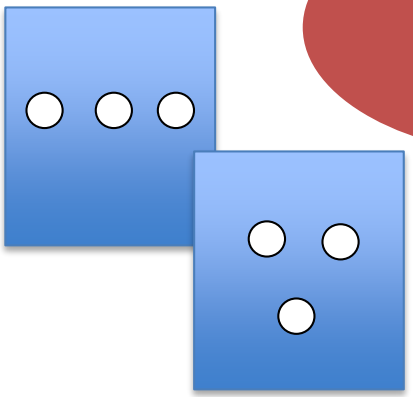
Brookes Publishing

Also Available from Amazon

Number Sense Development

Counting Skills

Quantity Connection



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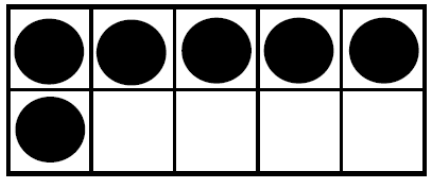

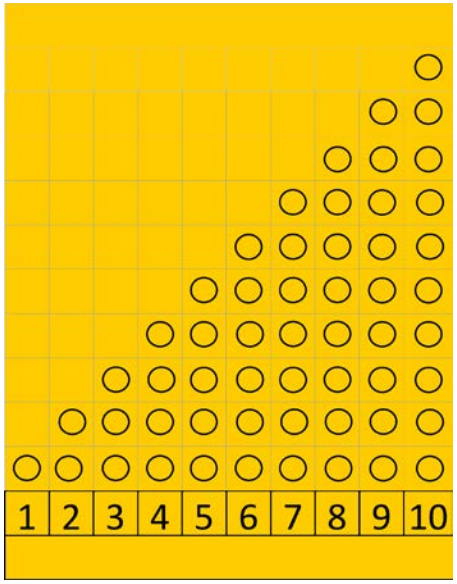

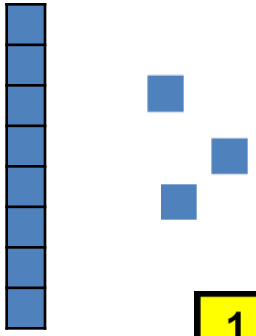

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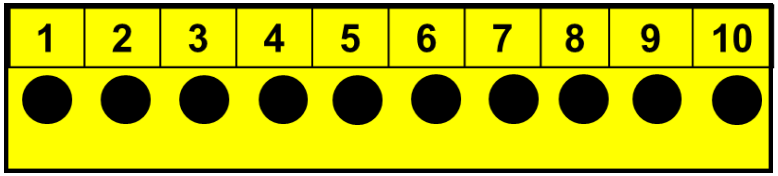
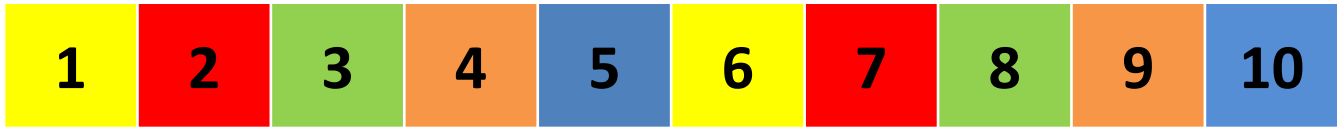


Number Sense Development

Magnitude Comparison

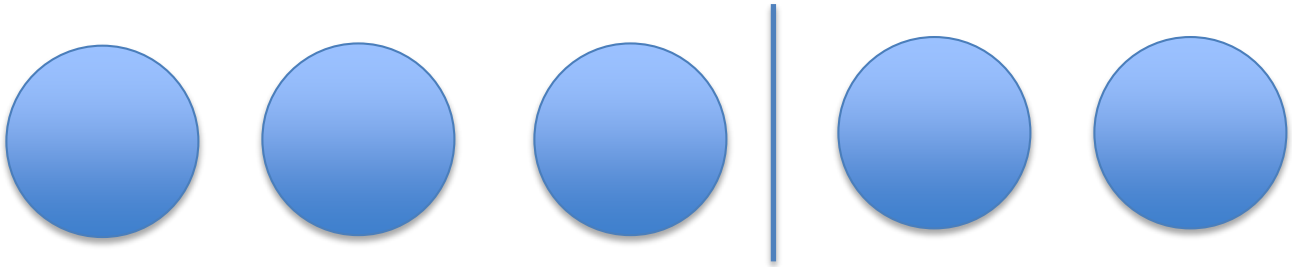
MENTAL NUMBER LIST

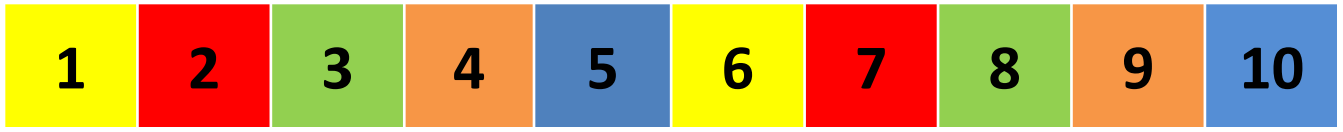


As you go up the list, the quantities get larger

Number Sense Development

Number Operations





3+2=5

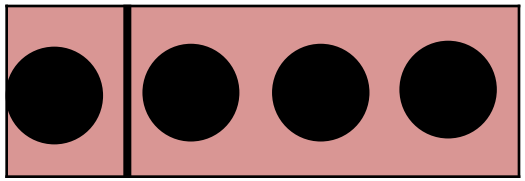
Number Sense Development

Number Operations

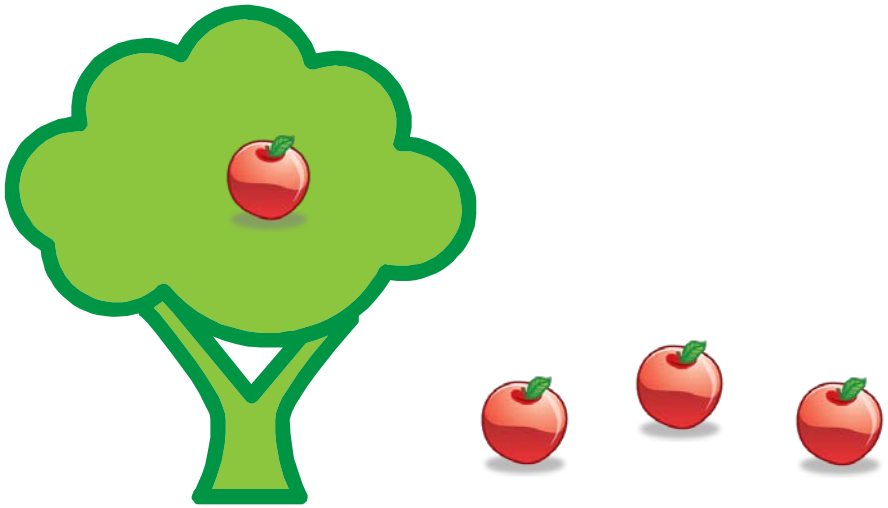
Combinations

1 + 3 = 4

Partners



Story problems

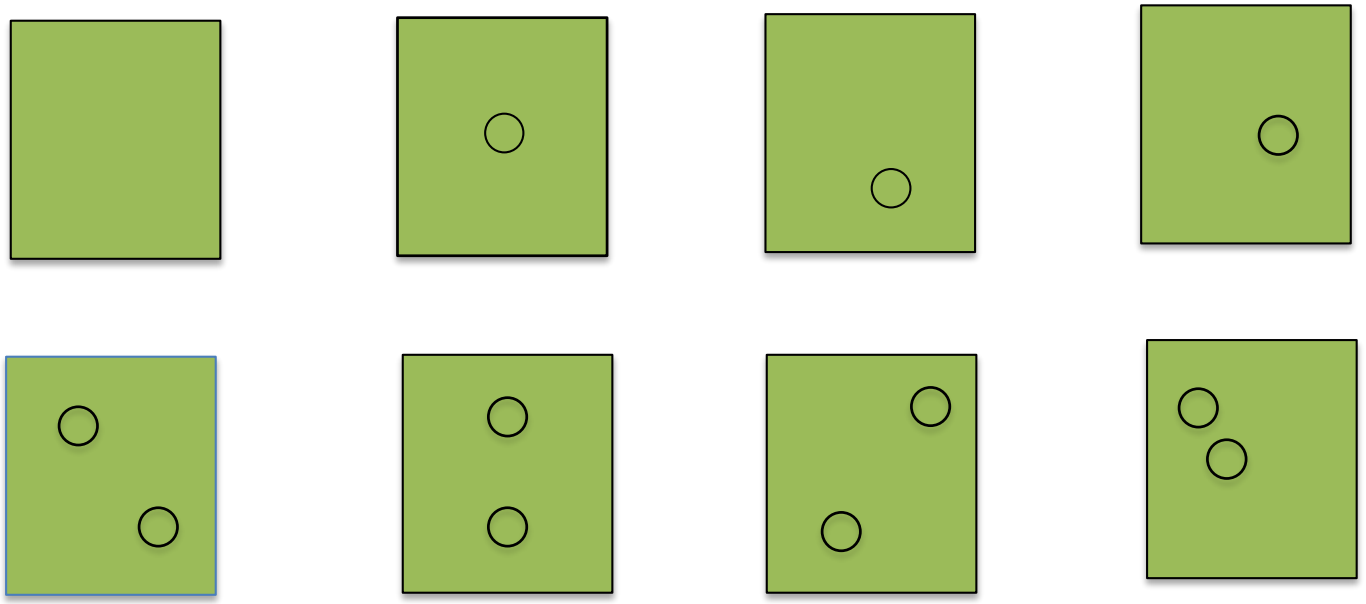


Intervention Study by Year

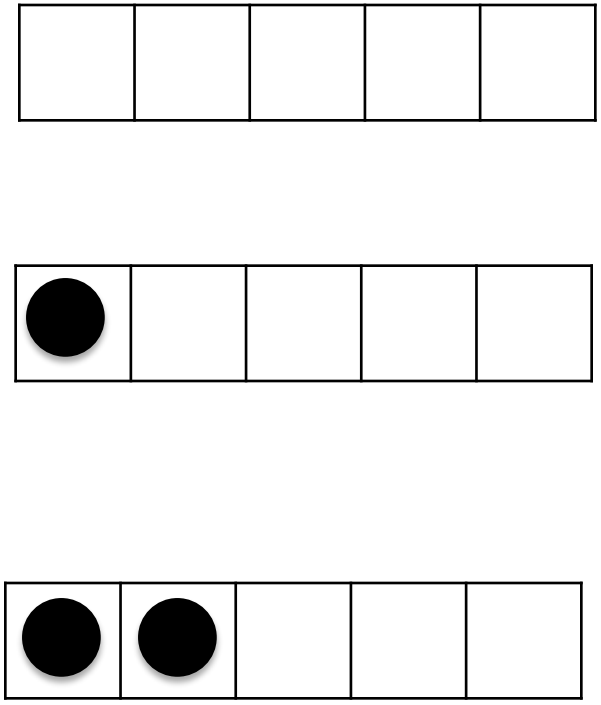
20 Lesson intervention - Each Lesson 30 minutes

Year one:	• Intervention successful with at-risk students • Schools serving low-income communities
Year two:	• Intervention successful with at-risk students • Schools serving low-income communities
Year three:	• Targeted at-risk students with low numeracy • Successful but effect sizes much smaller
Year four:	• Targeted at-risk students with low numeracy • Revised intervention • Successful with large effect sizes

Many Concepts - Few Numbers
0, 1, 2



SUBITIZING DOT CARDS



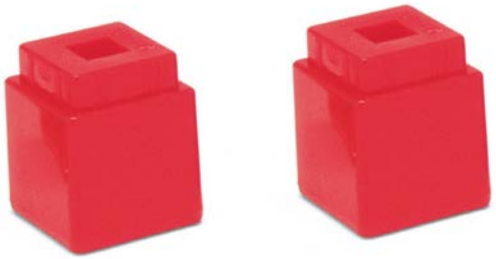
FIVE FRAMES

SUBITIZING FINGERS

QUANTITY CONNECTIONS

0

1

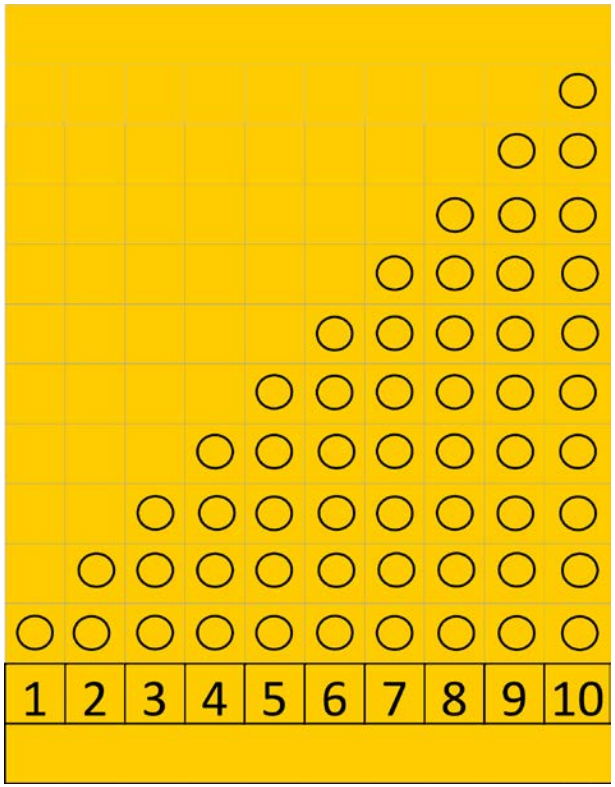


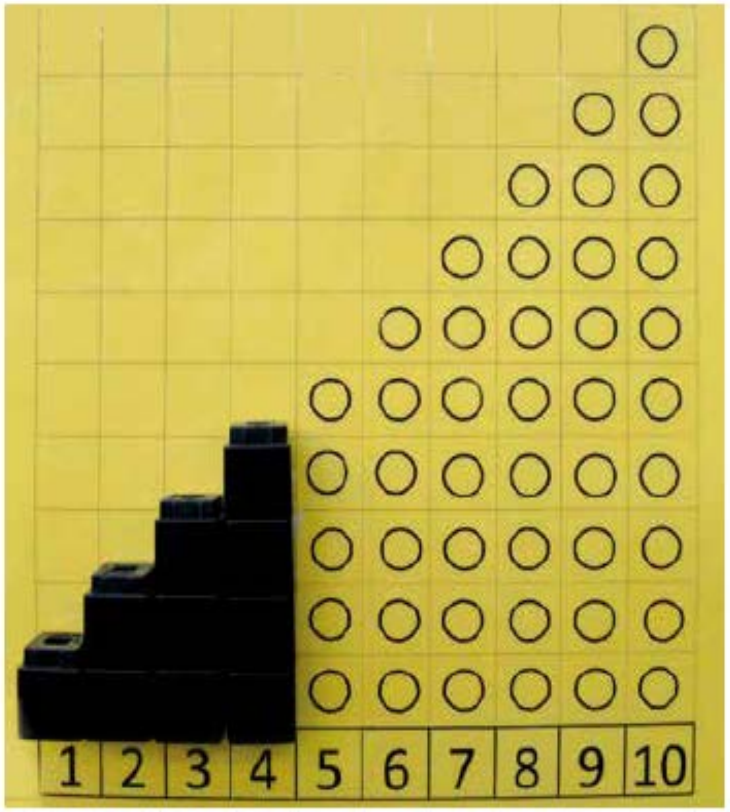
INTERLOCKING CUBES

2

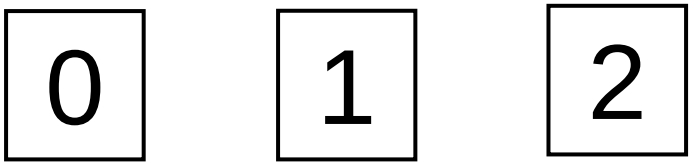
NUMBER RECOGNITION CARDS

MAGNITUDE COMPARISON



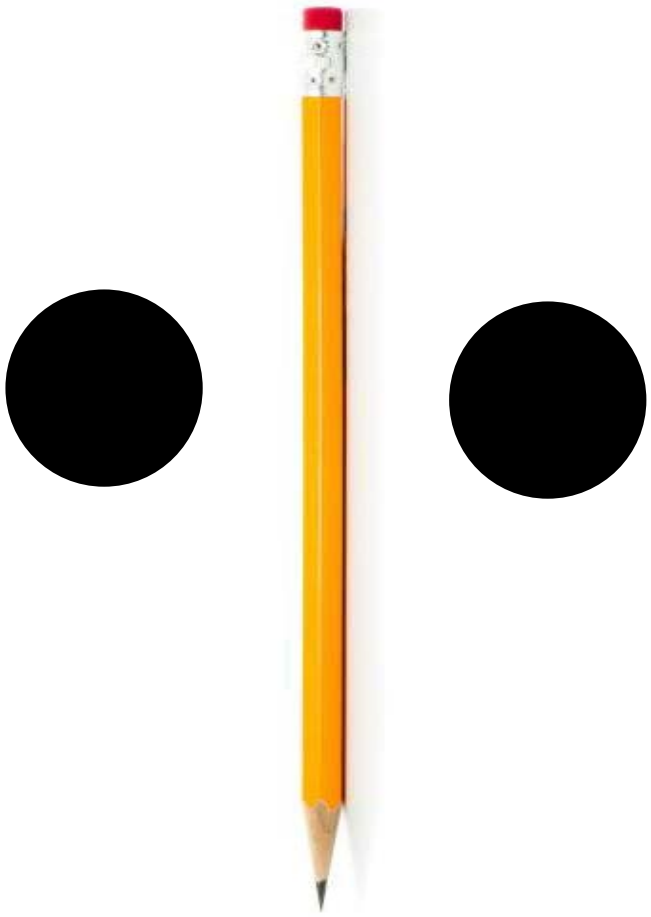


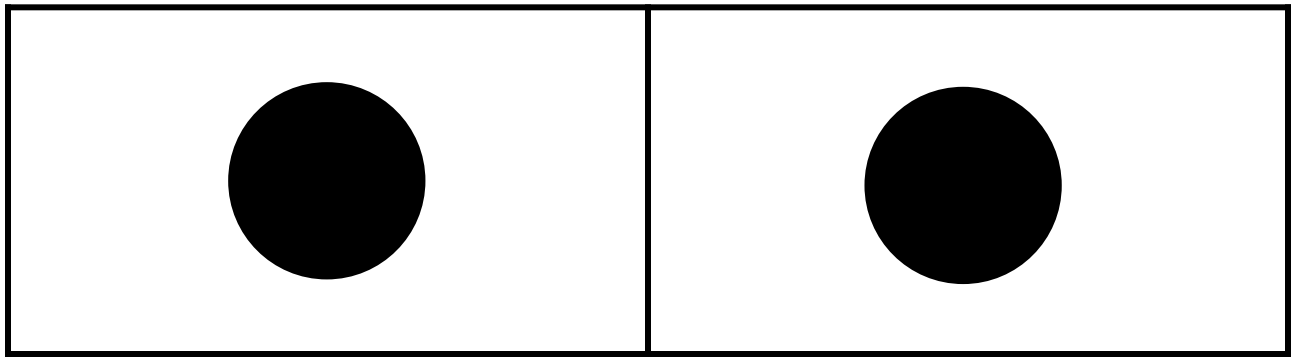
MAGNITUDE COMPARISON CARD GAMES

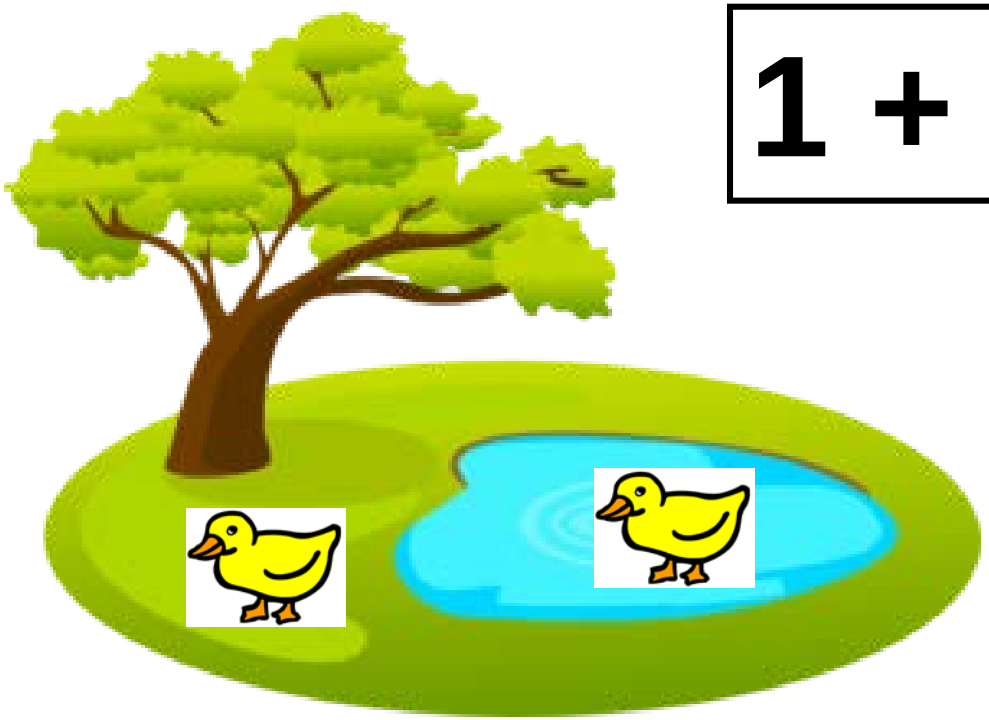


- Hold up two cards – “Which is Bigger?”
- Hold up one card – “What number comes before ___, what number comes after ____.”
- Put out all the cards – “Put these cards in order from smallest to largest.”

NUMBER OPERATIONS







$$1 + 1 = 2$$

$$2 - 1 = 1$$

INTRODUCING MORE NUMBERS

- Introduce one new number at a time
- Stop at 5 - ensure mastery before moving on
- Numbers 6 and above - 5 and 1,2,3,4,5
    - Fingers on 2 hands
    - Different color cubes above 5 on Cardinality Chart
    - Move to Ten Frame

INTRODUCING MORE NUMBERS

- Stop at 10 – ensure mastery
  - Move from Cardinality Chart to Number List
  - Play Number List Games
  - Before/After Games
  - Bigger Smaller

- Numbers above 10 – 20 10 and 1,2,3,4,5,6,7,8,9,10
  - Use cubes to create sticks of 10
  - Use stick of 10 *and* n single cubes to build the number
  - Use Decade and Unit cards to *build* the numeral

Center for Improving the Learning of Fractions

Principal Investigator: Dr. Nancy C. Jordan

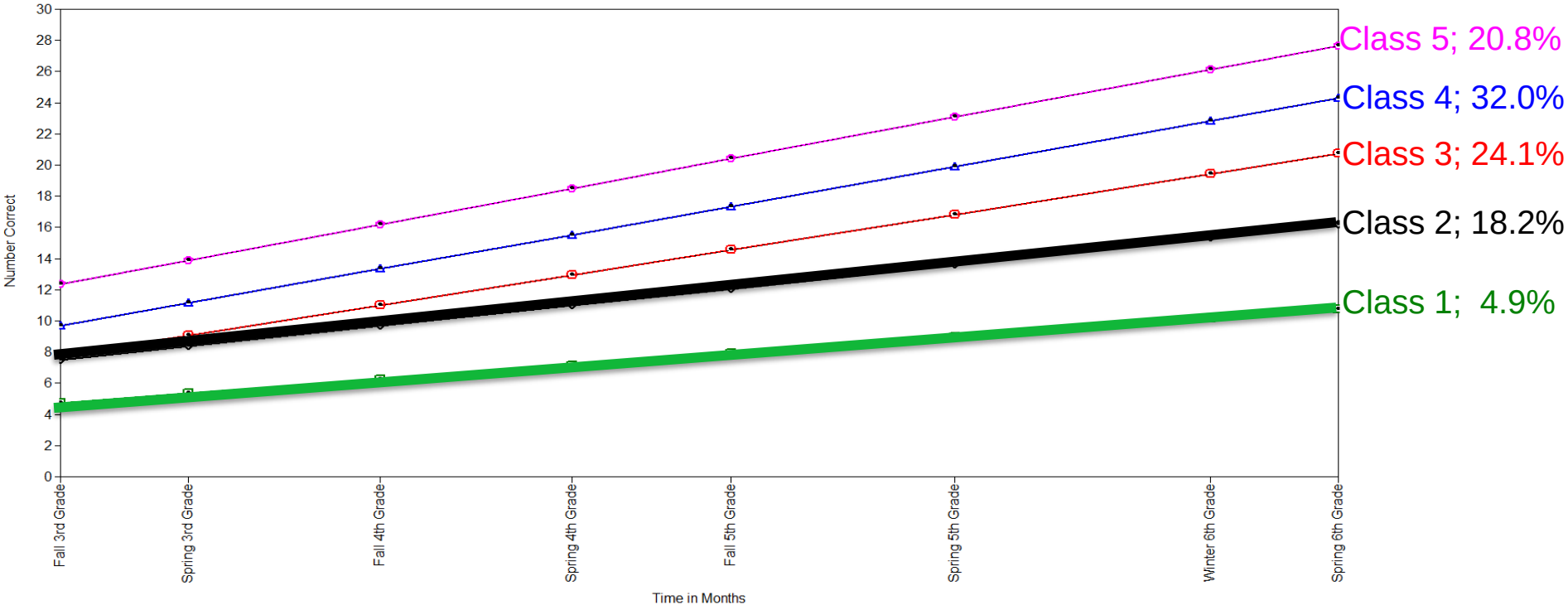
Goals of the Center:
Conduct longitudinal analyses from 3rd through 6th grade to…

- study the development of fraction knowledge
- examine predictors of fraction achievement
- assess how proficiency with whole number operations contribute to the ability to understand and operate with rational numbers.

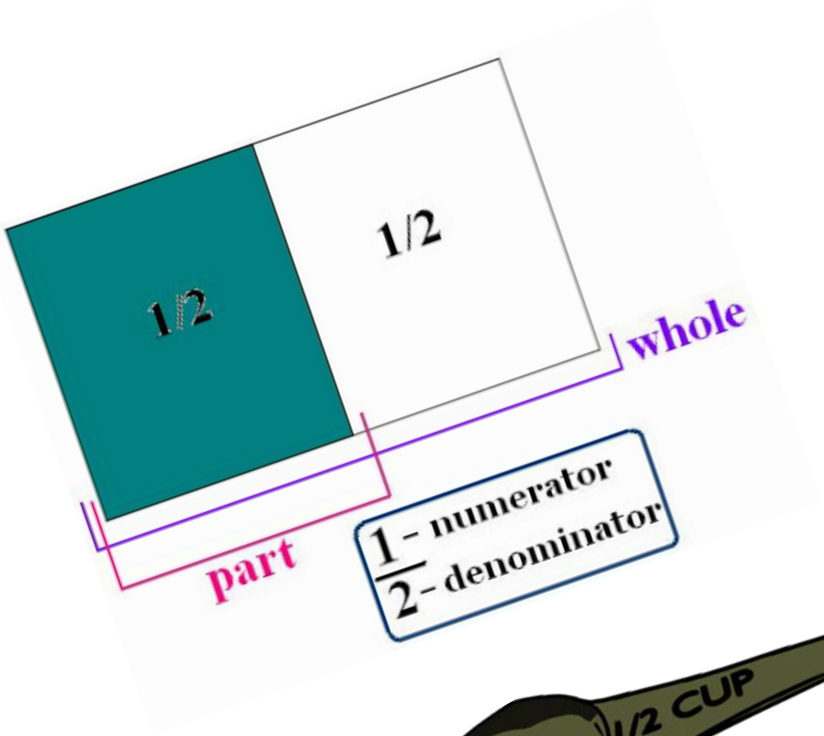
Latent Class Analysis

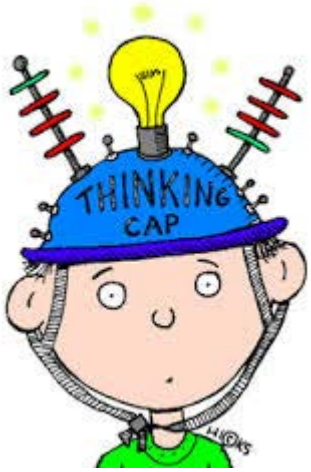
Fraction Concepts

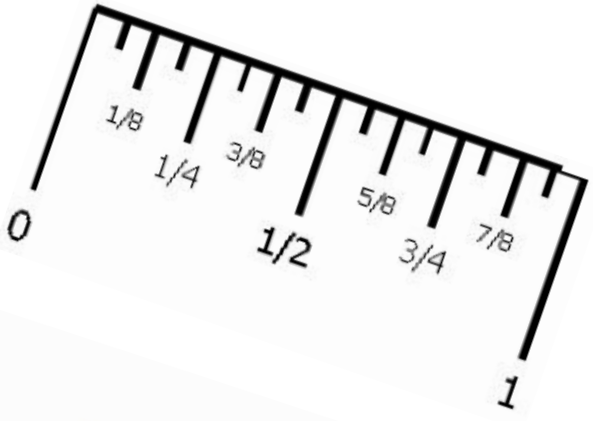
Hansen, N., Jordan, N. C., & Rodrigues, J. (2016)

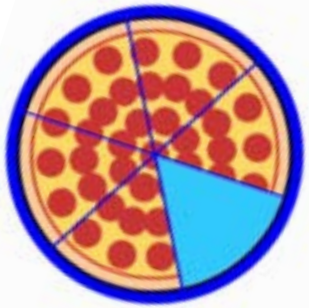


Developing a Fraction Sense Intervention for Struggling Middle School Students



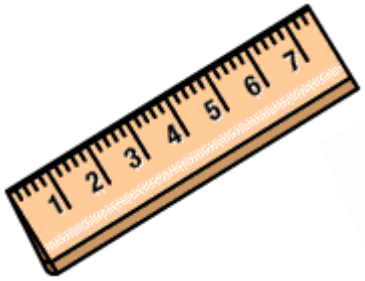






Fractions are Important!

Students will need fractions for every day problem solving




offthemark.com
THAT'S THE WRONG SHELF!
NO, IT'S NOT
HALF AND HALF
HALF AND HALF
WHOLE
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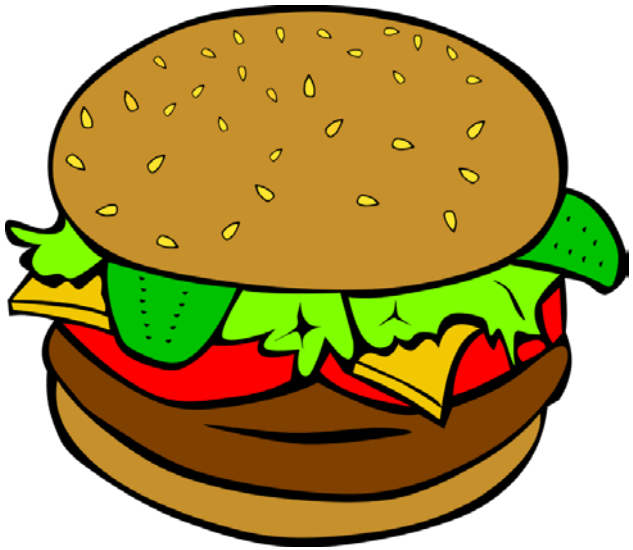
MarkParisi@aol.com
7-22
MARK PARISI
THE PROBLEM WITH HIRING A MATH MAJOR

Fractions do not make sense even to many adults!

$\frac{1}{3}$ pounder OR $\frac{1}{4}$ pounder

Which has more beef?

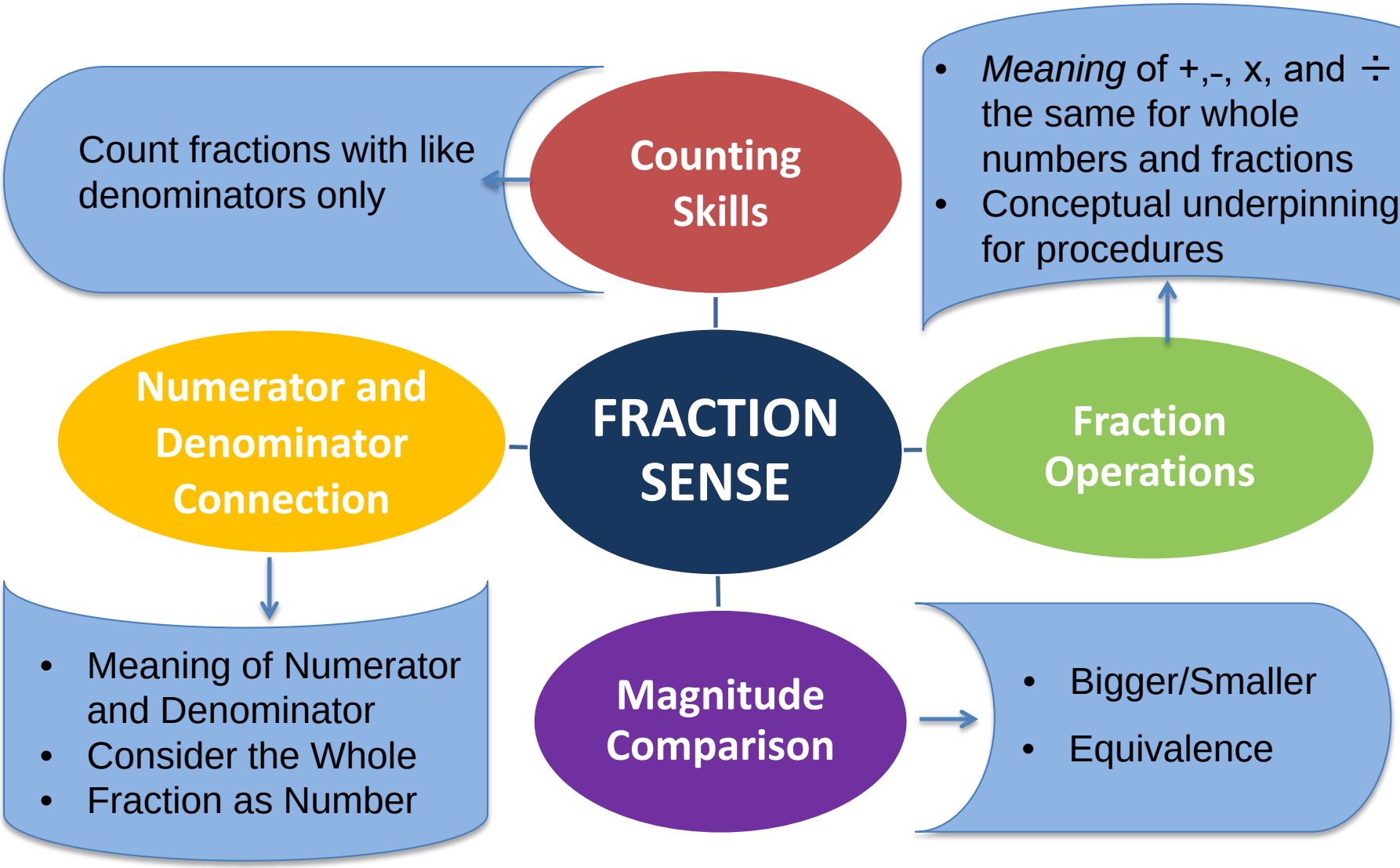


Why Fractions are Hard!

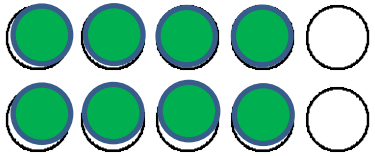
WHOLE NUMBERS	FRACTIONS
Only one numeral for each whole number magnitude	An infinite number of fractions can represent a given magnitude
0, 1, 2, 3, 4, ……	1/2 = 2/4 = 4/8 = 5/10 = ……
Each counting number is one more than the number before it.	There are an infinite number of fractions between any two numbers and the increment varies between fractions.
0+1=1, 1+1=2, 2+1=3, 3+1=4, …	
The magnitude of the number increases as you count higher.	The magnitude of a fraction increases as the numerator counts higher *(if the denominator stays the same).*
1< 2 < 3 < 4	. 1/4 < 2/4 < 3/4 < 4/4

Why Fractions are Hard!

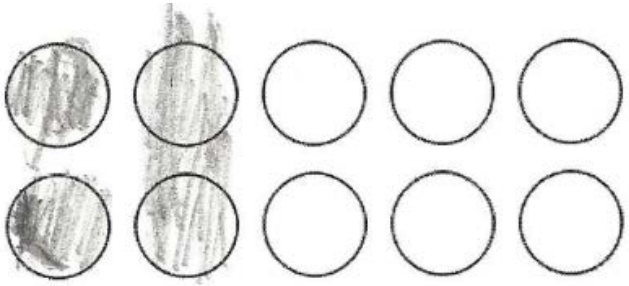
WHOLE NUMBERS	FRACTIONS
The magnitude of the number increases as you count higher.	The magnitude of a fraction *decreases* as the denominator counts higher (if the numerator stays the same).
1< 2 < 3 < 4	1/1 > 1/2 > 1/3 > 1/4
	But what about when they both change?!?
	2/4 < 3/5 3/5 < 2/3
The product of two whole numbers is always greater than (or equal to) either factor.	The product of two fractions can be less than either factor.
2 x 3 = 6	1/2 x 3/4 = 3/8


FRACTION SENSE
Counting Skills
Count fractions with like denominators only
Numerator and Denominator Connection
Meaning of Numerator and Denominator
Consider the Whole
Fraction as Number
Fraction Operations
Meaning of +,-, x, and ÷ is the same for whole numbers and fractions
Conceptual underpinnings for procedures
Magnitude Comparison
Bigger/Smaller
Equivalence

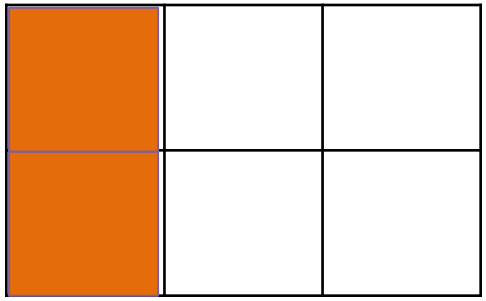
Examples of Misconceptions

$\frac{4}{5} =$ 

Most common incorrect response

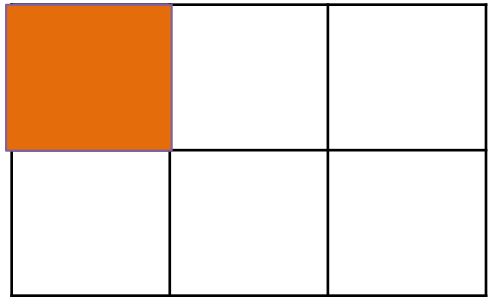
$\frac{4}{5} =$ 

Examples of Misconceptions



Shade $\frac{1}{3}$ of the rectangle above.

Most Common Incorrect Response:



Examples of Misconceptions

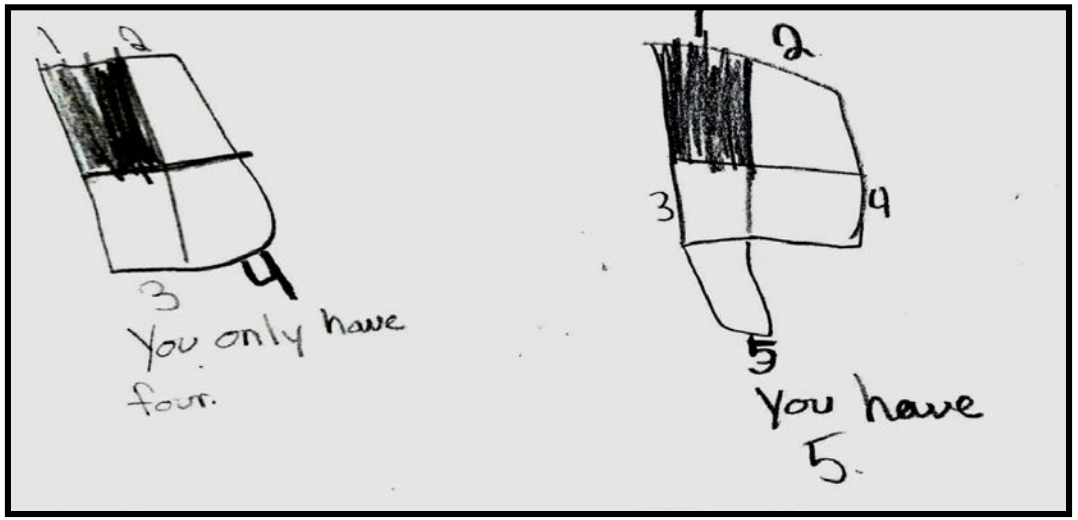
In which of the following are the three fractions arranged from least to greatest?

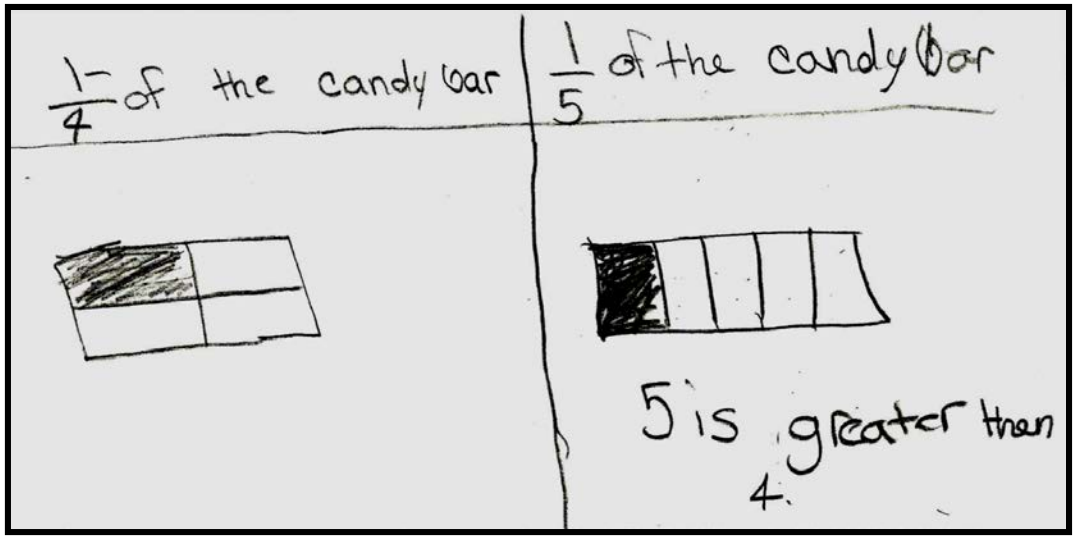
A.

Examples of Misconceptions

Mark says $\frac{1}{4}$ of his candy bar is smaller than $\frac{1}{5}$ of the same candy bar. Is Mark right? Explain.

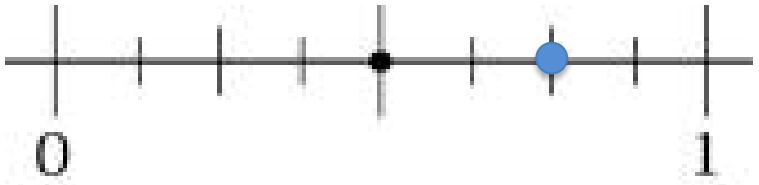
Comparing 1/4 and 1/5



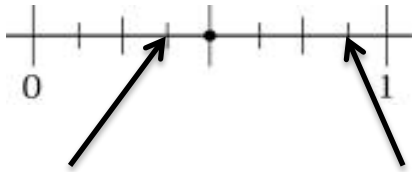


Examples of Misconceptions

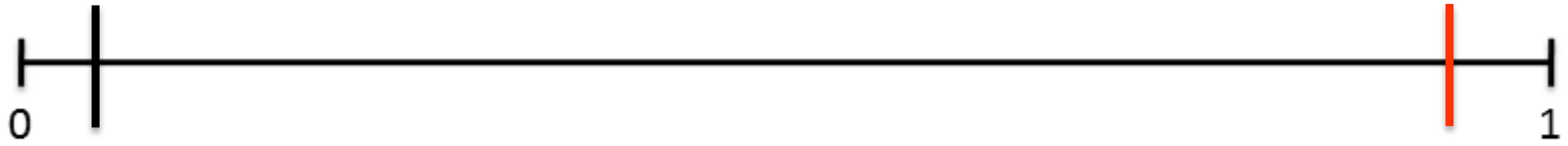
On the portion of the number line below, a dot shows where 1/2 is. Use another dot to show where **3/4** is.



Most Common Incorrect Responses:



Estimating fraction magnitudes on a number line:



$\frac{1}{19}$

$\frac{1}{2}$

Latent Class Analysis

Fraction Number Line Estimation

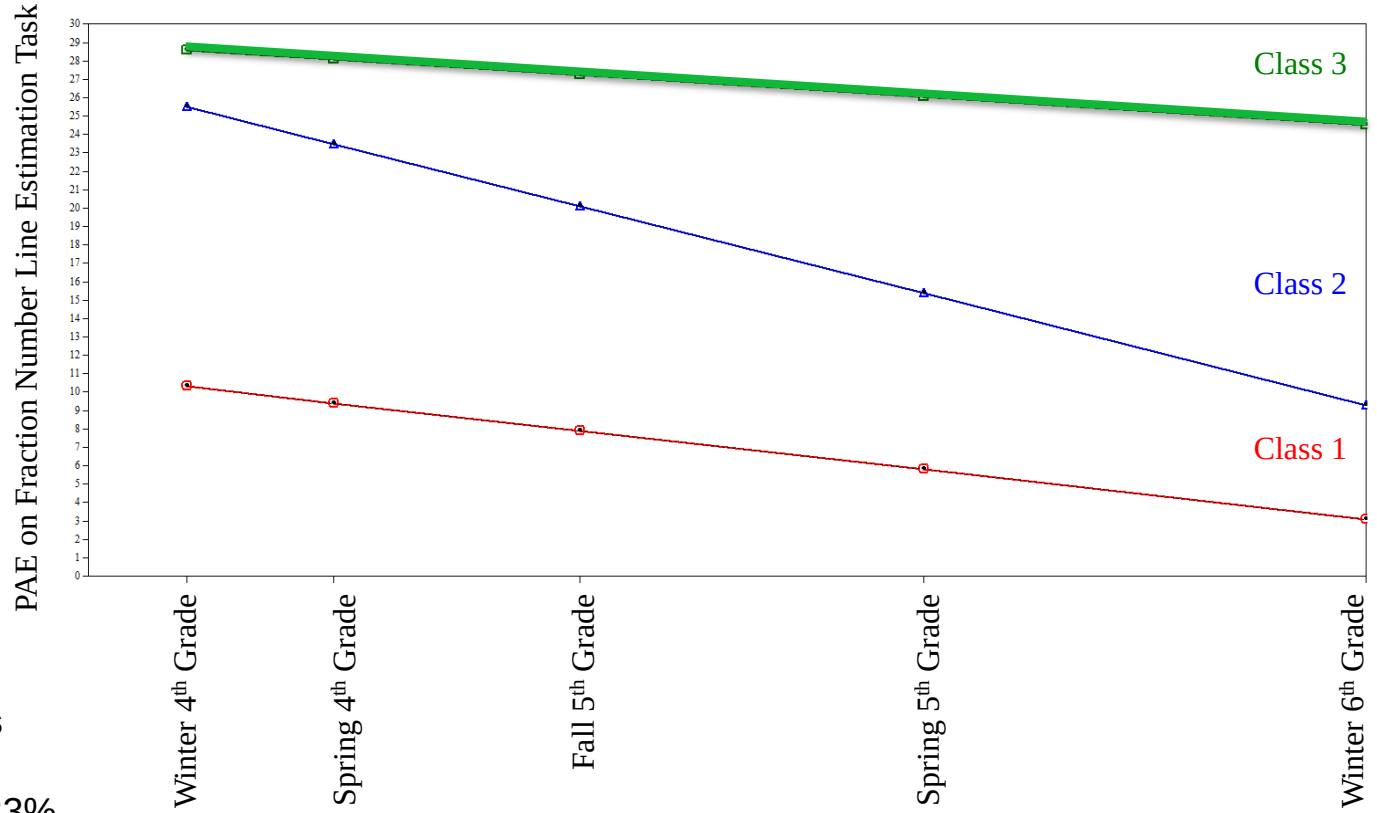
Resnick, I., Jordan, N.C., Hansen, et al. (2016)

Scores were calculated as Percent Absolute **Error (PAE)**, meaning that ***lower scores indicate better performance.***

Latent Class Analysis

Fraction Number Line Estimation

Resnick, I., Jordan, N.C., Hansen, et al. (2016)



FNLE Classes

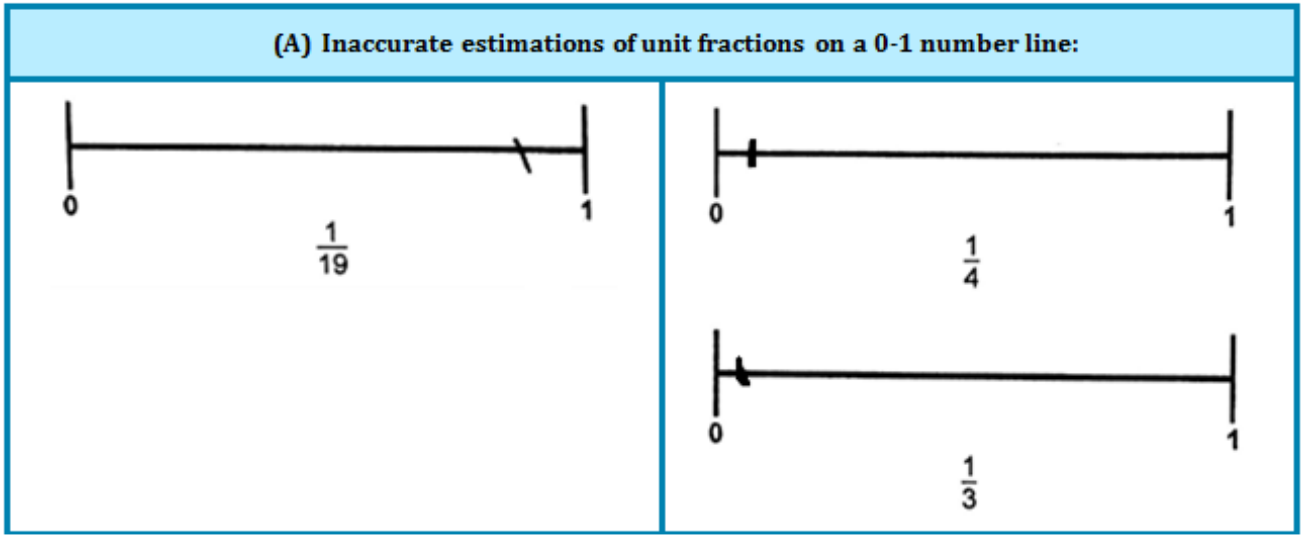
Total: 472

Class 1: 154; 33%

Class 2: 121; 25%

Class 3: 197; 42%

Examples of Misconceptions



Fraction Sense Intervention

Small group (4 students /1 instructor)
24 lessons, everyday
40 minutes each

COLOR RUN

Many Concepts - Few Numbers

HALVES

Developing Fraction Sense

40-minute lessons:

- Warm Up
- Exercises
- Huddle
- Fraction Game
- Sprint
- Cool Down

Developing Fraction Sense with Halves

WARM UP

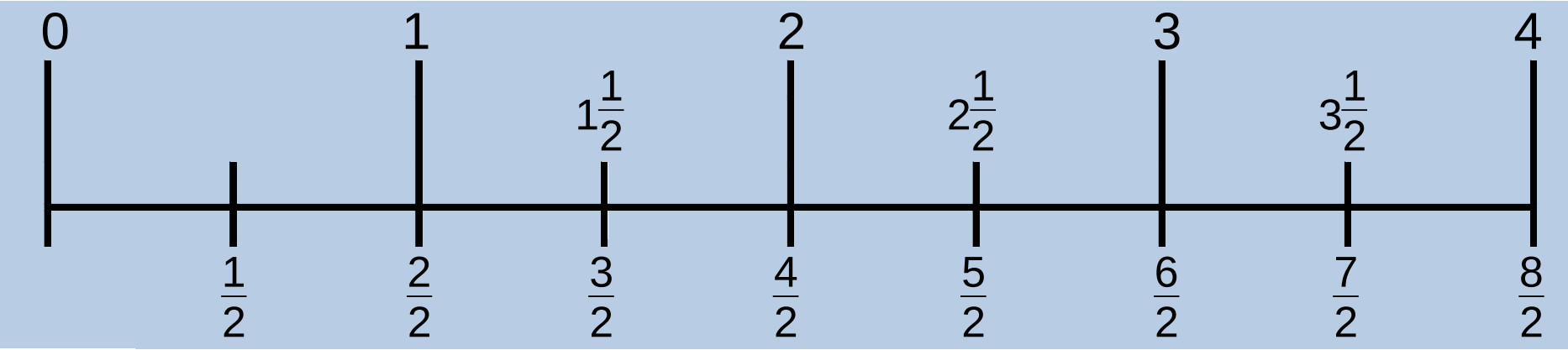
Written activity done as students arrive to the intervention.

Matches the previous day's **COOL DOWN**

Developing Fraction Sense with Halves

EXERCISES

Oral Counting Activities with Halves



Developing Fraction Sense with Halves

HUDDLE

Develop Fraction Concepts, Vocabulary, and Strategies using Halves

- 2 EQUAL Portions ?
- Ask, “Of What Whole?”

Developing Fraction Sense with Halves

$\frac{1}{2}$ of what WHOLE

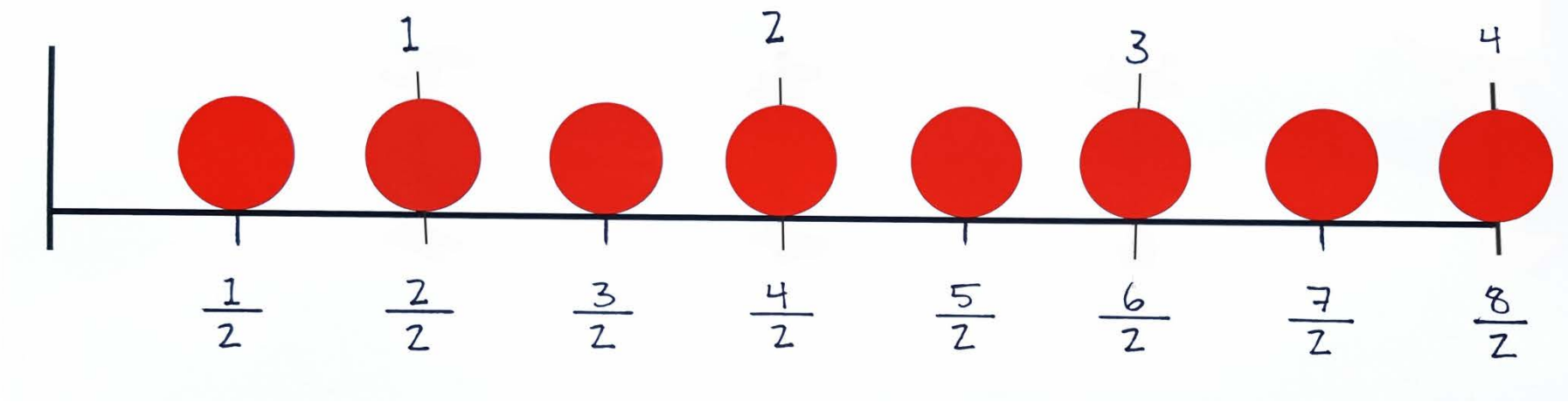

1 Whole
1/2
1/2

Developing Fraction Sense with Halves

Developing Number Line Concepts Using a Race Course

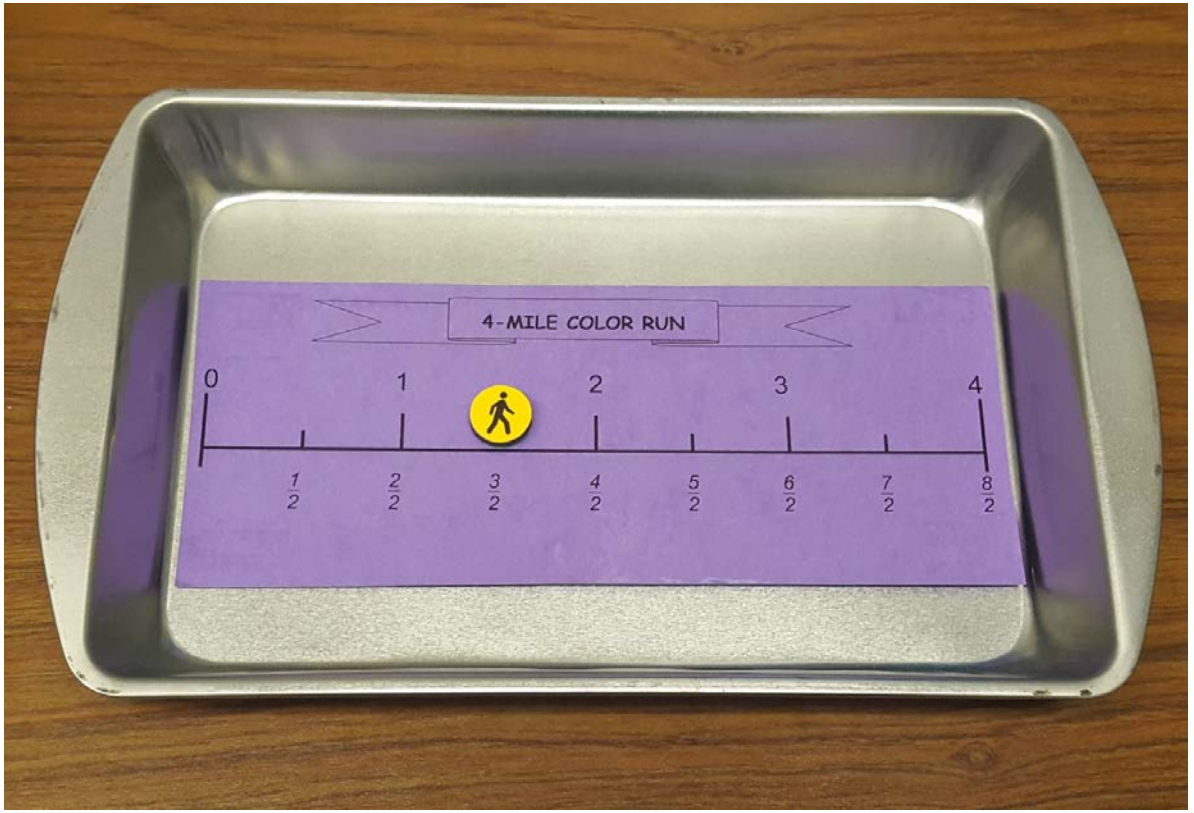
Use a paper bar to measure whole miles

4-Mile Race Course with Half Miles Marked



1. Mark Halves (use folded paper bar to check accuracy)
2. Put on stickers
3. Label halves – note whole number/halves equivalencies

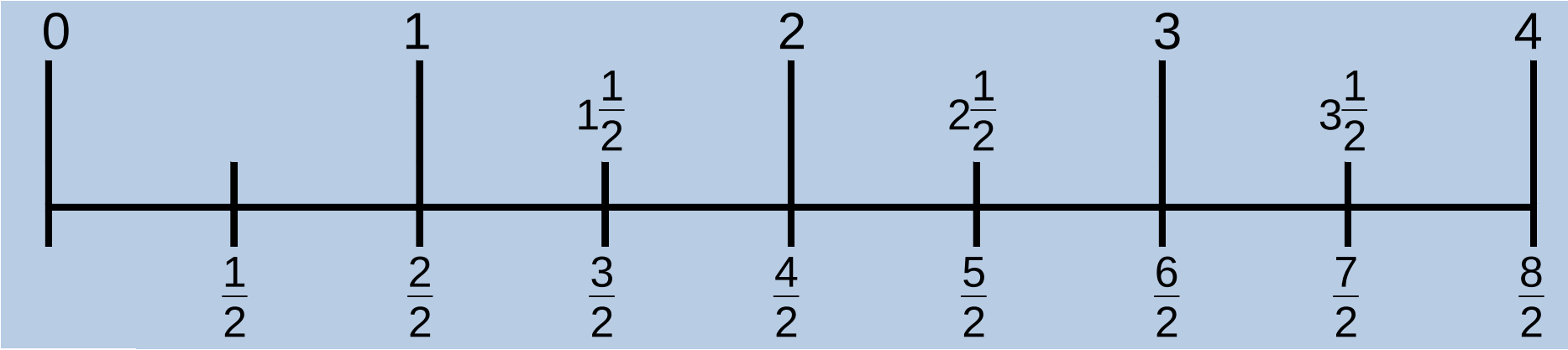
Walking the Race Course to Solve Equations



2/2 + 1/2 = 3/2

3/2 - 1/2 = 2/2

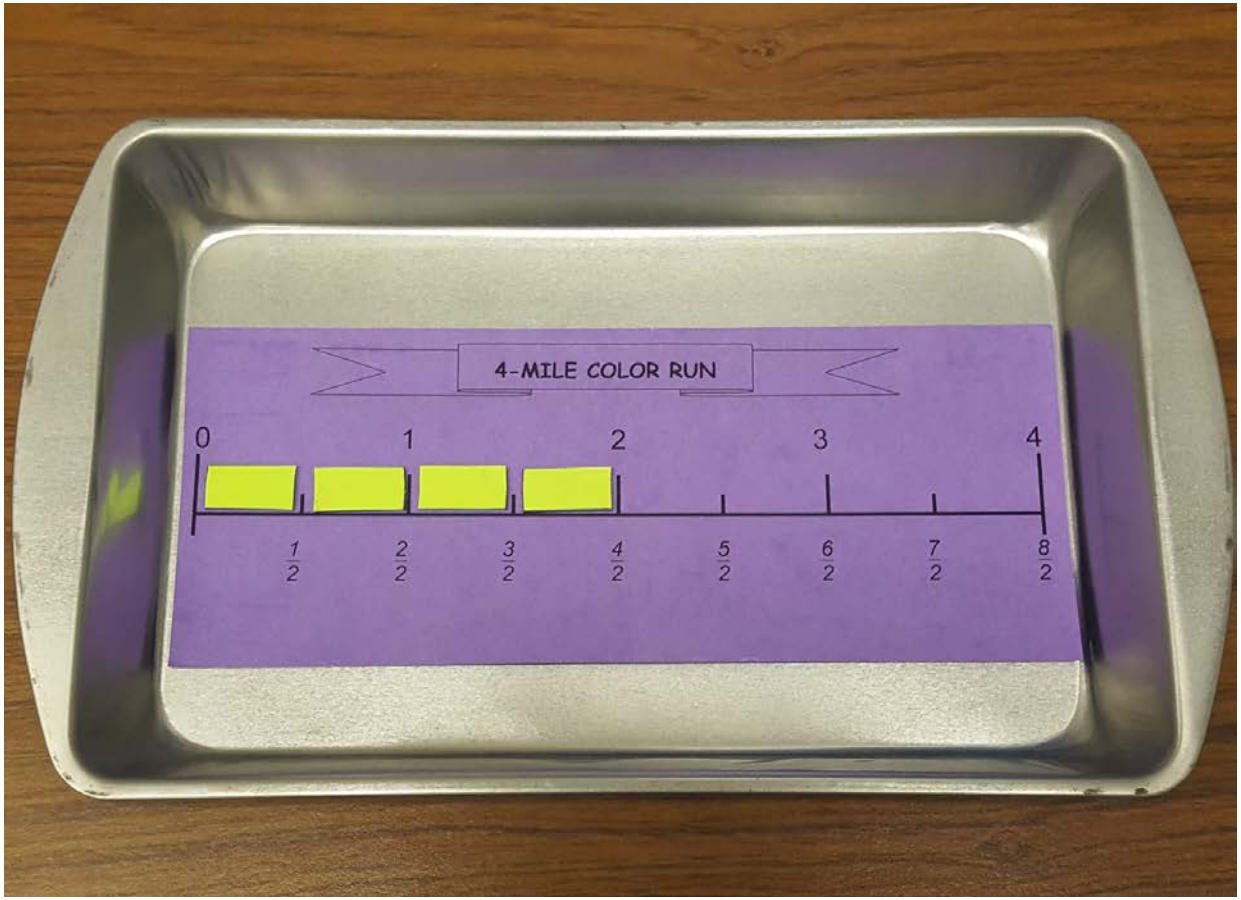
Whole and Mixed Numbers



- Note Mixed Number / Improper Fraction Equivalences
- Change From Mixed Number to Improper Fraction
- Change from Improper Fraction to Mixed Number

Connecting Fraction Models to Deepen Conceptual Understanding

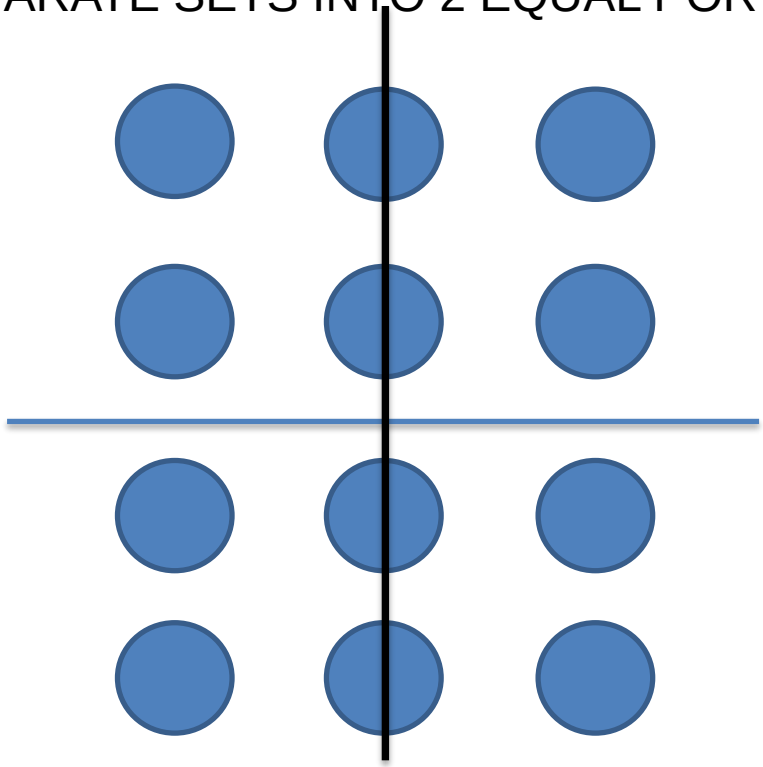
Using fraction bars on the number line emphasizes that ½ stands for the distance from 0 to ½ - not just the location where the ½ is written.



USING SAME STRATEGY for FRACTION of a SET

Find ½ of 12 circles

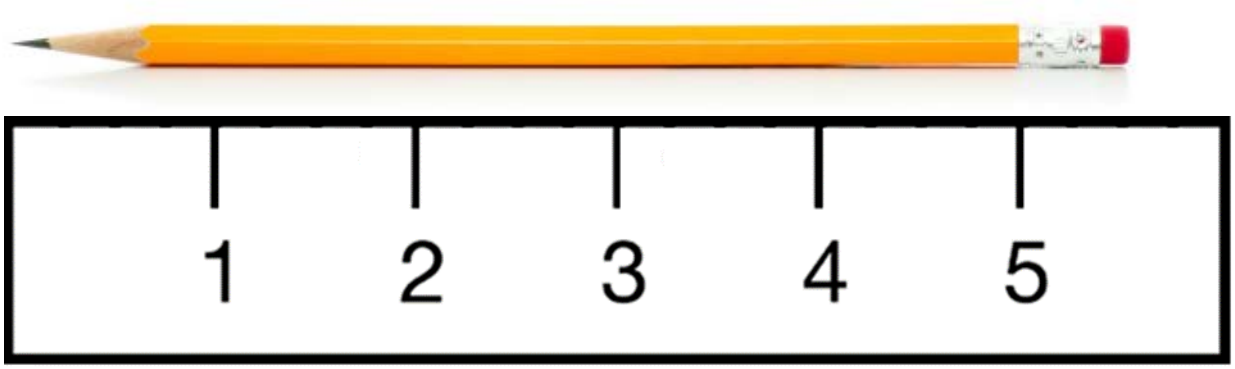
SEPARATE SETS INTO 2 EQUAL PORTIONS



½ of 12 circles is 6 circles (introduction to multiplication by a fraction)

CONNECTING TO EVERYDAY ACTIVITIES

RULER ACTIVITIES
Ruler Marked in Halves



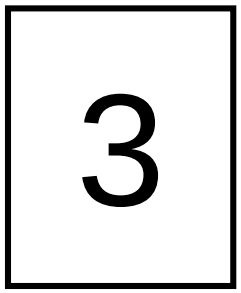
Introducing THIRDS with a Proportional Reasoning Task

A Measuring Cup = a *Vertical* Number Line

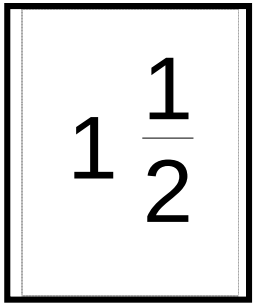
Developing Fraction Sense with Halves

FRACTION GAMES

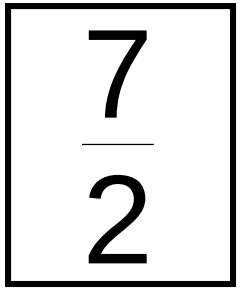
Fast-paced card games to reinforce lesson concepts and develop fluency



How many halves?



How many halves?



What mixed number?

Developing Fraction Sense with Halves

SPRINT

Multiplication Facts that Support the Lesson

For Halves: n x 2

5 x 2 = 10

Initial Practice with Front of Card

5 x 2 =

Final Practice with Back of Card

Front of card is shown for ERROR correction – students always end with a correct statement

Developing Fraction Sense with Halves

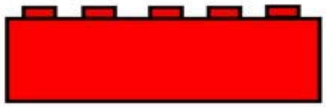
COOL DOWN

Formative assessment and review

Use your ruler to measure the length of the pen to the **exact** $\frac{1}{2}$ **inch**:

_____in.

Use your ruler to measure the length of the Lego piece to the **closest** $\frac{1}{2}$ **inch**:



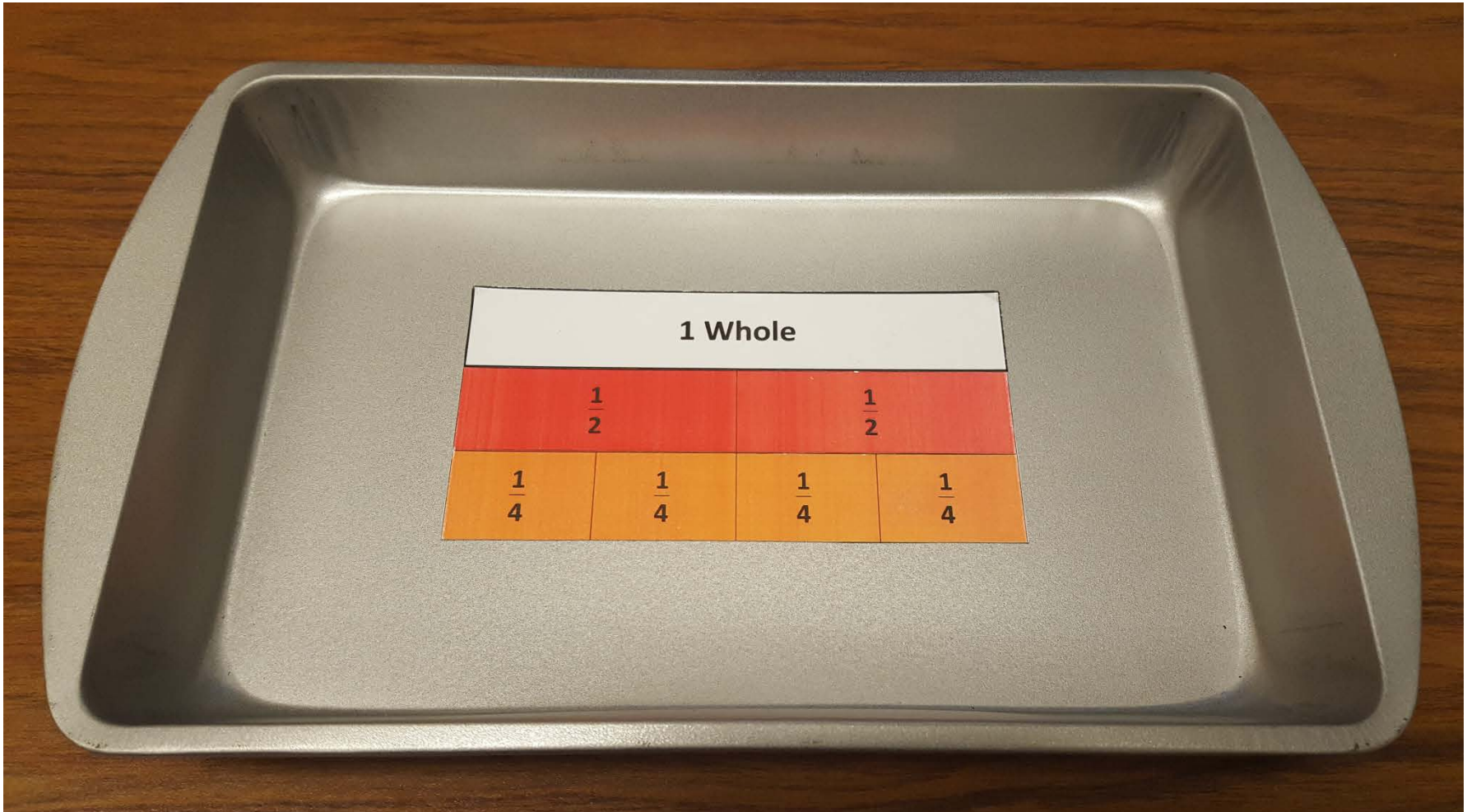
_____in.

Use your rulers to help you solve the equations.
Write your answers with **mixed** or **whole numbers**.

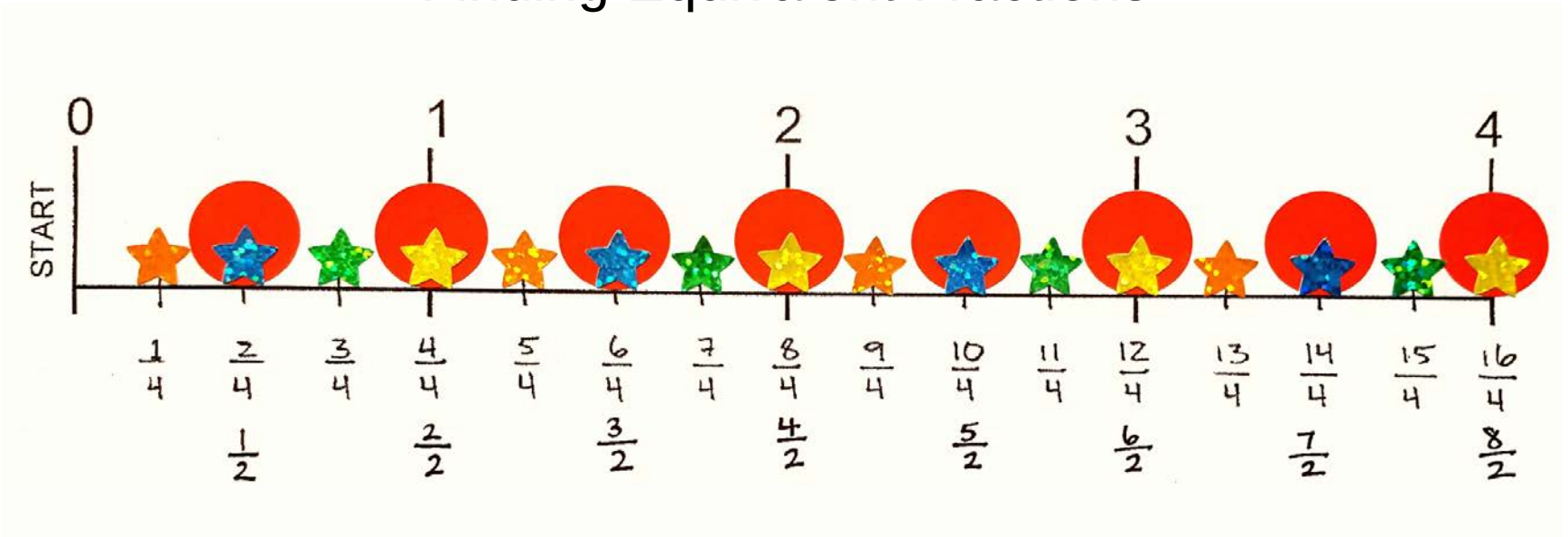
$\frac{5}{2}$ inches + $\frac{2}{2}$ inches =

$\frac{4}{2}$ inches - $\frac{3}{2}$ inches =

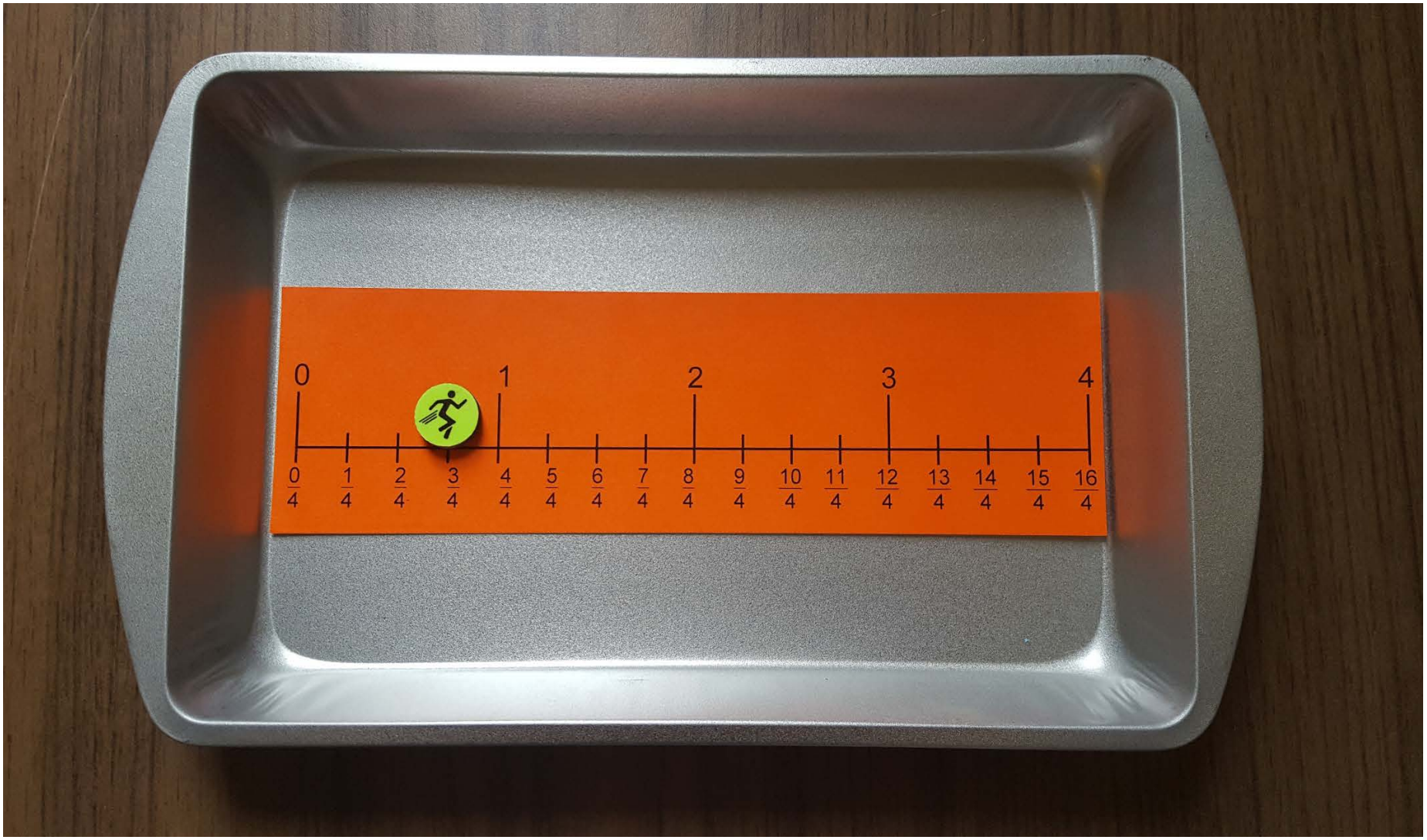
SEPARATING HALVES TO MAKE FOURTHS



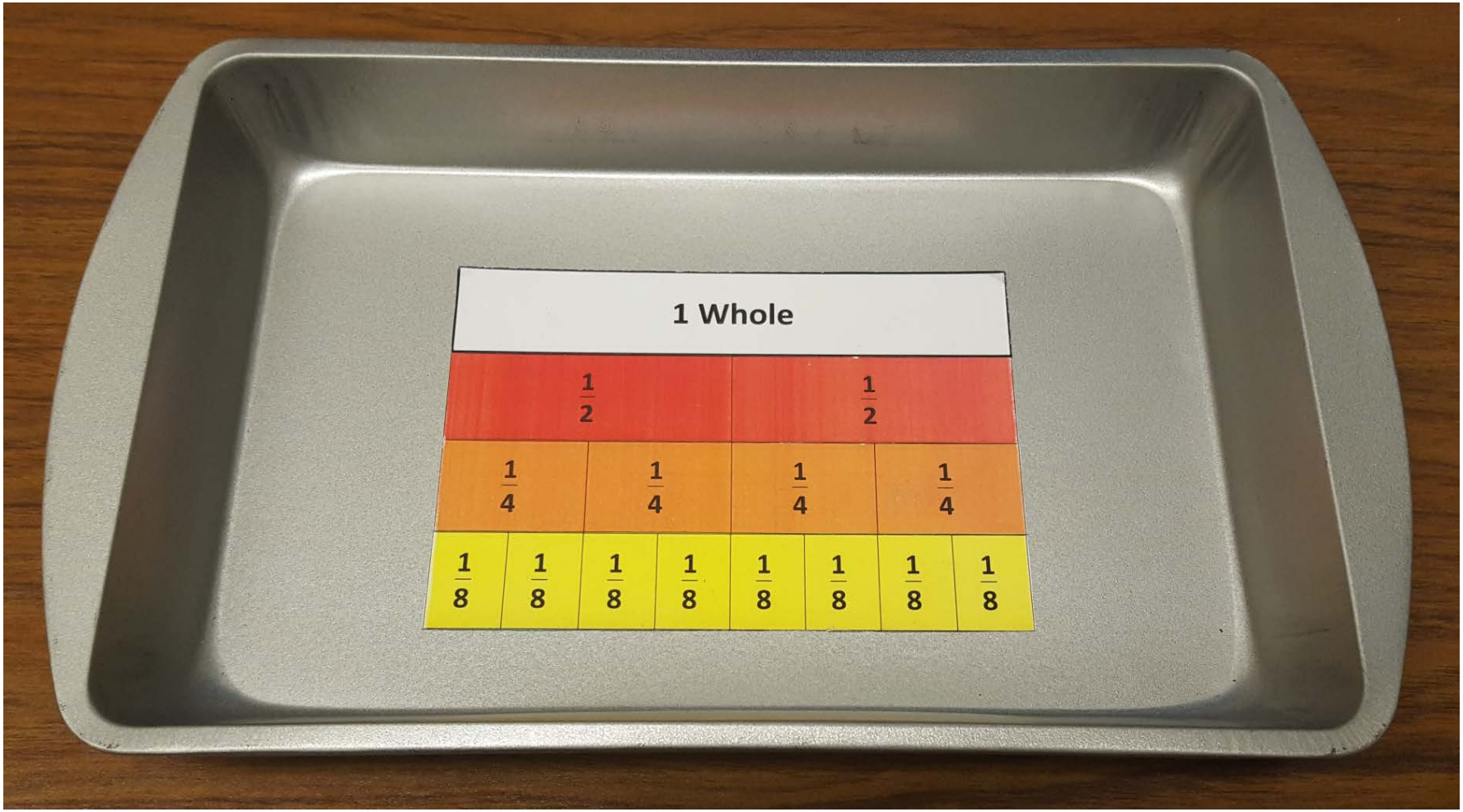
SEPARATING HALVES TO MAKE FOURTHS
Finding Equivalent Fractions



- Mark Halves – put on stickers
- Mark Fourths – put on stickers
- Label Fourths
- Label Halves
- Find Equivalent Fractions


0
1
2
3
4
0/4 1/4 2/4 3/4 4/4 5/4 6/4 7/4 8/4 9/4 10/4 11/4 12/4 13/4 14/4 15/4 16/4

AND SO FORTH……..



Teaching Exceptional Children

Preparing for Algebra by Building Fraction Sense

Jessica Rodrigues, Nancy I. Dyson, Nicole Hansen, and Nancy C. Jordan

THANK YOU!

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