NCTM Innov8
Some Research-Affirmed Practical Strategies for Making Math Far More Accessible

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In a nutshell, we know what it takes

What we strive for: LEARNING SUCCESS

The enabling conditions: ENGAGEMENT
PARTICIPATION

What it takes: TASKS QUESTIONS DISCOURSE
MISTAKES PRODUCTIVE STRUGGLE

Enhanced by: REPRESENTATIONS
ALTERNATIVES
ESTIMATES

All guided by: A CLARITY OF GOALS

Let’s begin with some glimpses of what it can and should look like.

What's the question? What's the answer?

Omaha
244 miles

Car Manual
Gas Tank Capacity:
21 gallons

Average MPG:
28.3

- **Why did you care?**
- **Will you make it?**
- **About how far will you get?**
- **How much gas needs to be in the tank to be sure you make it?**

- **What made this more accessible?**

Grade 6 SA Harlem Central Tues Dec 8, 2015

- Lesson 6 in the Expressions unit (6.EE standards)
- Ally and Mabubar co-teaching
- 19 Scholars
- Driven by a number strings mini-lesson, a Math Workshop task and an exit ticket
- “Our goal for today is to “identify, create and understand equivalent expressions.”
- “Zayasia, can you please repeat our learning goal?”
- “Let’s begin with out number strings.”

Number strings for today's Mini Lesson

Are they equivalent? How do you know?

1. **4(8) = 4(3 + 5)**
2. **4(8) = 4(a + 5)**
3. **4(8) = 4(3 + b)**
4. **3x + 3y = 3(x + y)**

Let's summarize: For each: Always, sometimes, never equivalent?

Math Workshop Task

Jan normally rides her bike to and from work.

Her normal route is 18 miles from home to work.

One day she goes to a coffee shop on her way to work and on her way home.

This adds x miles to her trip each way.

(“What do you notice?” “What’s the question?”)

Great: Write and show the distance Jan travels using a diagram or picture and two different, but equivalent, expressions.

Resulting in:

18	x
x	18

- 18 + x + 18 **+ x**
- **2(18 + x)**
- **2(x + 18)**
- **x + x + 18 + 18**
- **36 + 2x**

“Is everyone correct? Turn and tell your partner why?”
“What do the numbers and variables represent?”
“Which expression is simplest or easiest to use? Why?”

Lesson 6 Exit Ticket

Which of the following represent equivalent expressions?

Explain or show your process of determining which expressions ARE equivalent.

Select all that apply:

a. $x + x + x + x = 4x$

b. $15y + 5x = 3(5y + x)$

c. $6(2 + x) = 12 + 6$

d. $3(x + y) = 3x + y$

One by one, just consider how each of these 8 MTPs live in this lesson?

- **Establish mathematics goals to focus learning.**
- **Implement tasks that promote reasoning and problem solving.**
- **Use and connect mathematical representations.**
- **Facilitate meaningful mathematical discourse.**
- **Pose purposeful questions.**
- **Build procedural fluency from conceptual understanding.**
- **Support productive struggle in learning mathematics.**
- **Elicit and use evidence of student thinking.**

Big Idea:

Answer getting

Vs.

Explanations, alternatives, connections

Again: What made this more accessible?

So let's look at some of these characteristics in greater depth.

Join me in Teachers' Chat Room

- They forget
- They don't see it my way
- They approach it differently
- They don't follow directions
- They give ridiculous answers
- They don't remember the vocabulary
- They ask why do we need to learn this

THEY THEY THEY BLAME BLAME BLAME

An achievement gap or an INSTRUCTION gap?

Well…..if…..

- **They forget – so we need to more deliberately review;**
- **They see it differently – so we need to accommodate multiple representations;**
- **They approach it differently – so we need to elicit, value and celebrate alternative approaches;**
- **They give ridiculous answers – so we need to focus on number sense and estimation;**
- **They don’t understand the vocabulary – so we need to build language rich classrooms;**
- **They ask why do we need to know this – so we need to embed the math in contexts.**

Yes

But how?

Number from 1 to 6

- 1. What is 6 x 7?
- 2. What number is 1000 less than 18,294?
- 3. About how much is 32¢ and 29¢?
- 4. What is 1/10 of 450?
- 5. Draw a picture of 1 2/3
- 6. About how big is this room?

Strategy #1

Incorporate on-going cumulative review into instruction every day.

Implementing Strategy #1

Almost no one masters something new after one or two lessons and one or two homework assignments. That is why one of the most effective strategies for fostering mastery and retention of critical skills is daily, cumulative review at the beginning of every lesson.

On the way to school:

- **A fact of the day**
- **A term of the day**
- **A picture of the day**
- **An estimate of the day**
- **A skill of the day**
- **A measurement of the day**
- **A word problem of the day**

Or in 2nd grade:

- How much bigger is 9 than 5?
- What number is the same as 5 tens and 7 ones?
- What number is 10 less than 83?
- Draw a four-sided figure and all of its diagonals.
- About how long is this pen in centimeters?

Good morning Boys and Girls Number from 1 to 5

1. What is the value of $\tan(\pi/4)$?
2. Sketch the graph of $(x-3)^2 + (y+2)^2 = 16$
3. What are the equations of the asymptotes of $f(x) = (x-3)/(x-2)$?
4. If $\log_2 x = -4$, what is the value of x?
5. About how much do I weight in kg?

Consider how we teach reading:
JANE WENT TO THE STORE.

- **Who went to the store?**
- **Where did Jane go?**
- **Why do you think Jane went to the store?**
- **Do you think it made sense for Jane to go to the store?**

Now consider mathematics:

TAKE OUT YOUR HOMEWORK.

#1 19

#2 37.5

#3 185

(No why? No how do you know? No who has a different answer?)

Strategy #2

Adapt from what we know about reading

(incorporate literal, inferential, and evaluative comprehension to develop stronger neural connections)

SALE

Pencils 3¢

Pens 4¢

Limit of 2 of each!

I need a pencil and a pen.

How much? How did you get it? Why did you do that?

Look at the power of our questions!

Straight from the text:

Jan has picked 2605 apples. She has 91 boxes. How many apples will Jan put in each box if each box holds the same number of apples?

UGH!

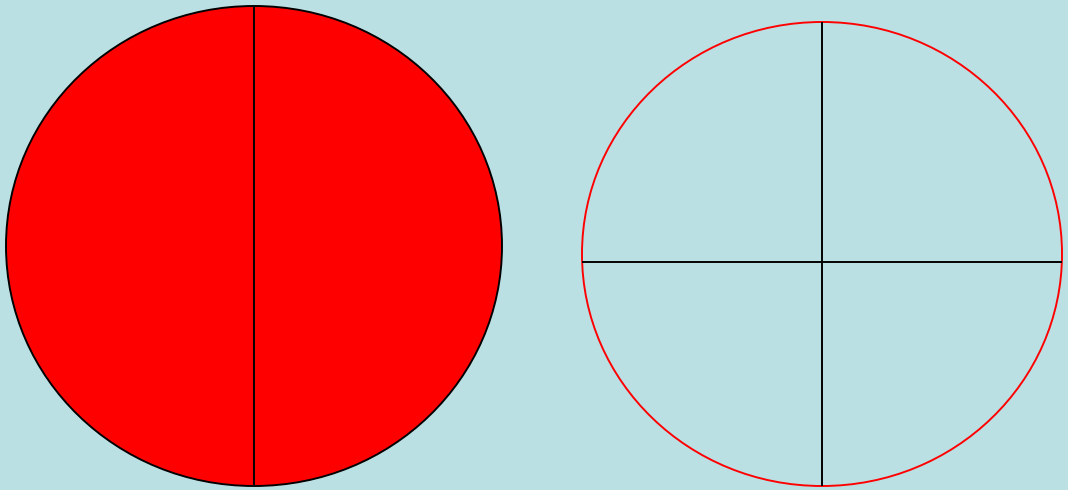
Adapting what the text bestows

Sarah has 91 empty boxes.

She had 2605 apples to pack into the boxes.

- So what does the 91 tell us, 2605?
- What do you think the question is?
- About how many apples do you think would be in each box? More than 100? Less than 100? Convince us
- Can you draw a picture?
- Can you create a number sentence?
- Why did you divide and not multiply?
- So how many apples would be in each box if…

Tell me what you see.



Tell me what you see.

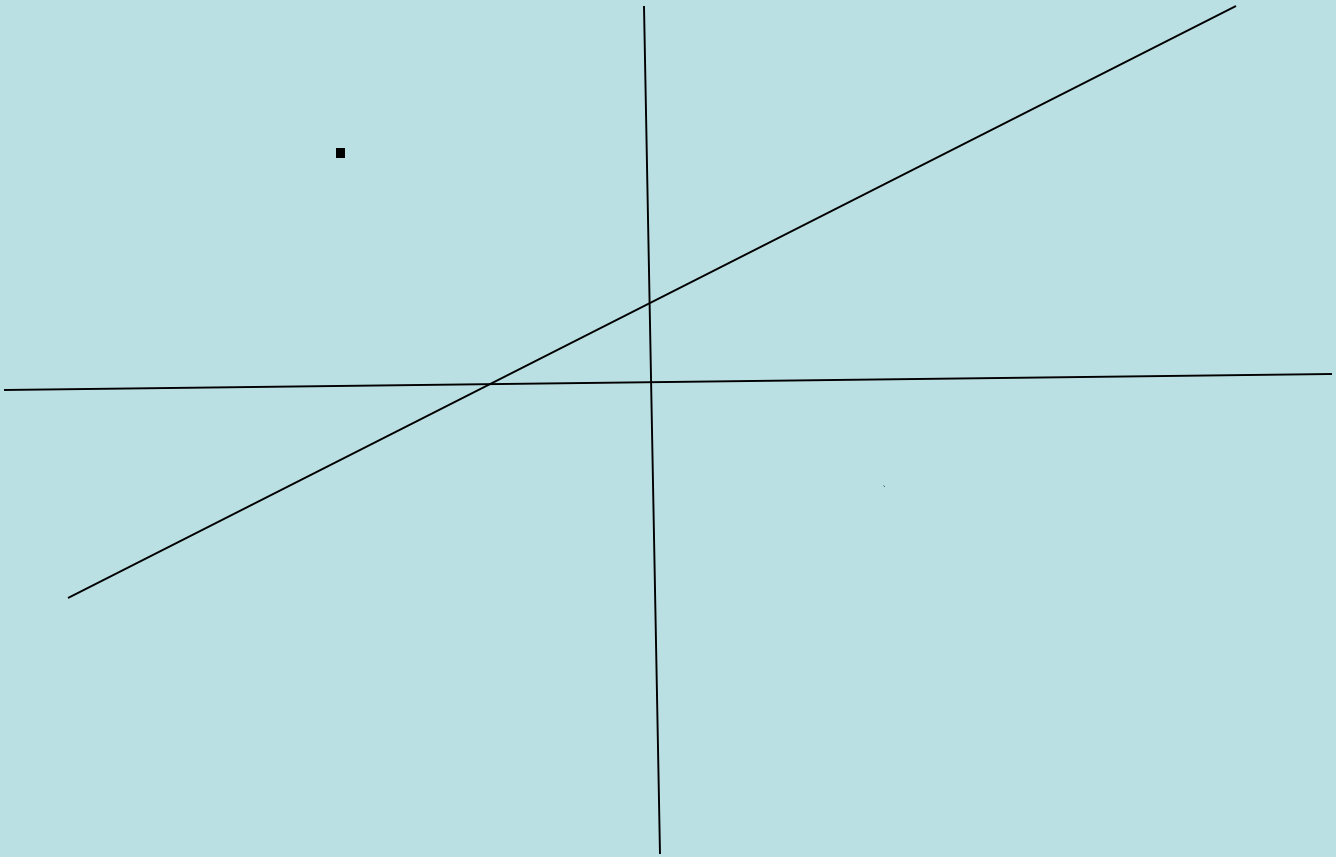
73

63

Tell me what you see.

2 1/4

Tell me what you see.



Tell me what you see.

$$f(x) = x^2 + 3x - 5$$

Strategy #3

Create a language rich classroom.

(Vocabulary, terms, answers, explanations)

Implementing Strategy #3

Like all languages, mathematics must be encountered orally and in writing. Like all vocabulary, mathematical terms must be used again and again in context and linked to more familiar words until they become internalized.

Area = covering

Perimeter = border

Cos = bucket

Circumference = a belt

Quotient = sharing

Mg = grain of sand

Cubic = S

Surface area = skin

Tan = sin/cos = y/x for all points on the unit circle

Ready, set, picture…..
“three quarters”
Picture it a different way.

Why does this make a difference?
Consider the different ways of thinking about the same mathematics:

- 2 ½ + 1 ¾
- $2.50 + $1.75
- 2 ½” + 1 ¾”

Show me 20 centimeters with your hands.

Strategy #4

Draw pictures/
Create mental images/
Foster visualization

The power of models and representations

Siti packs her clothes into a suitcase and it weighs 29 kg.

Rahim packs his clothes into an identical suitcase and it weighs 11 kg.

Siti's clothes are three times as heavy as Rahims.

What is the mass of Rahim's clothes?

What is the mass of the suitcase?

The old (only) way:

Let S = the weight of Siti's clothes

Let R = the weight of Rahim's clothes

Let X = the weight of the suitcase

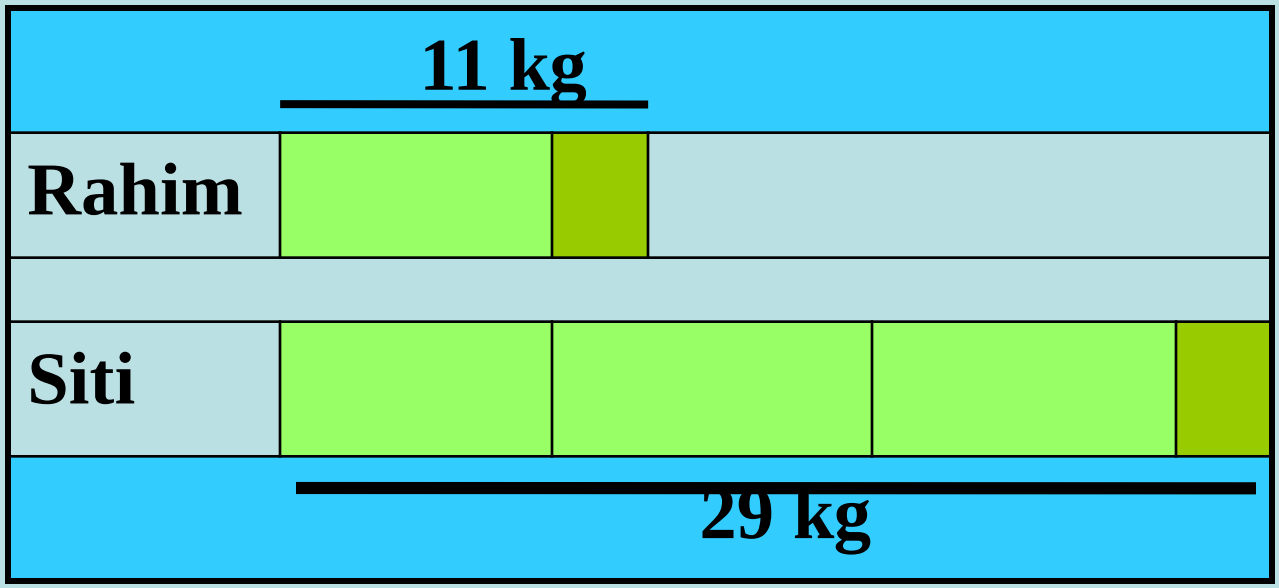
$$S = 3R \qquad S + X = 29 \qquad R + X = 11$$

so by substitution: $3R + X = 29$

and by subtraction: $2R = 18$

so $R = 9$ and $X = 2$

Or using a model:



You choose:

3 + 4 = 10 - 3 = 2 x 4 = etc.

Vs.

SALE

Pencils 3¢

Pens 4¢

Limit of 2 of each!

My Store

SALE

Pencils 3¢

Pens 4¢

Erasers 5¢

Limit of 3 of each!

SO?

Your turn

Pencils 7¢

Pens 8 ¢

Erasers 9 ¢

Limit of 10 of each.

I just spent 83 ¢ (no tax) in this store.

What did I purchase?

Pencil	**7¢**	**0**		**1**	**3**	**3**	**2**	**1**	**0**	**8**	
Pen	**8¢**	**0**				**1**	**3**	**5**	**7**	**0**	
Eraser	**9¢**	**10**	**9**	**8**	**7**	**6**	**5**	**4**	**3**	**3**	
		83¢									

You choose:

$$\begin{array}{r} 85 \\ -\ 47 \\ \hline \end{array}$$

vs.

I've got $85. You've got $47.

SO?

You choose:

1.59) 10

vs.

You have $10. Big Macs cost $1.59

SO?

You choose….

- **The one right way to get the one right answer that no one cares about and isn't even asked on the state tests**

vs.

- **Where am I? (the McDonalds context)**
- **Ten? Convince me.**
- **About how many? How do you know?**
- **Exactly how many? How do you know?**
- **Oops – On sale for $1.29 and I have $20.**

You choose:

Given: F = 4 (S – 65) + 10

Find F when S = 81.

Vs.

The speeding fine in Vermont is $4 for every mile per hour over the 65 mph limit plus $10 handling fee.

Connecticut: F = 10 (S – 55) + 40

Maximum speeding fine: $350

- **Describe the fine in words**
- **At what speed does it no longer matter?**
- **At 80 mph how much better off would you be in VT than in CT?**
- **Use a graph to show this difference**

For example:

Solve for x: $16 \times .75^{x} < 1$

Vs.

You ingest 16 mg of a controlled substance at 8 a.m. Your body metabolizes 25% of the substance every hour. Will you pass a 4 p.m. drug test that requires a level of less than 1 mg? At what time could you first pass the test?

Which class do YOU want to be in?

Strategy #5

Embed the mathematics in contexts;

Present the mathematics as problem situations.

Implementing Strategy #5

Here's the math I need to teach.

When and where do normal human beings encounter this math?

Last and most powerfully:

Make “why?”
“how do you know?”
“convince me”
“explain that please”
your classroom mantras

To recapitulate:

1. Incorporate on-going cumulative review
2. Parallel literal to inferential to evaluative comprehension used in reading
3. Create a language-rich classroom
4. Draw pictures/create mental images
5. Embed the math in contexts/problems

And always ask them "why?"

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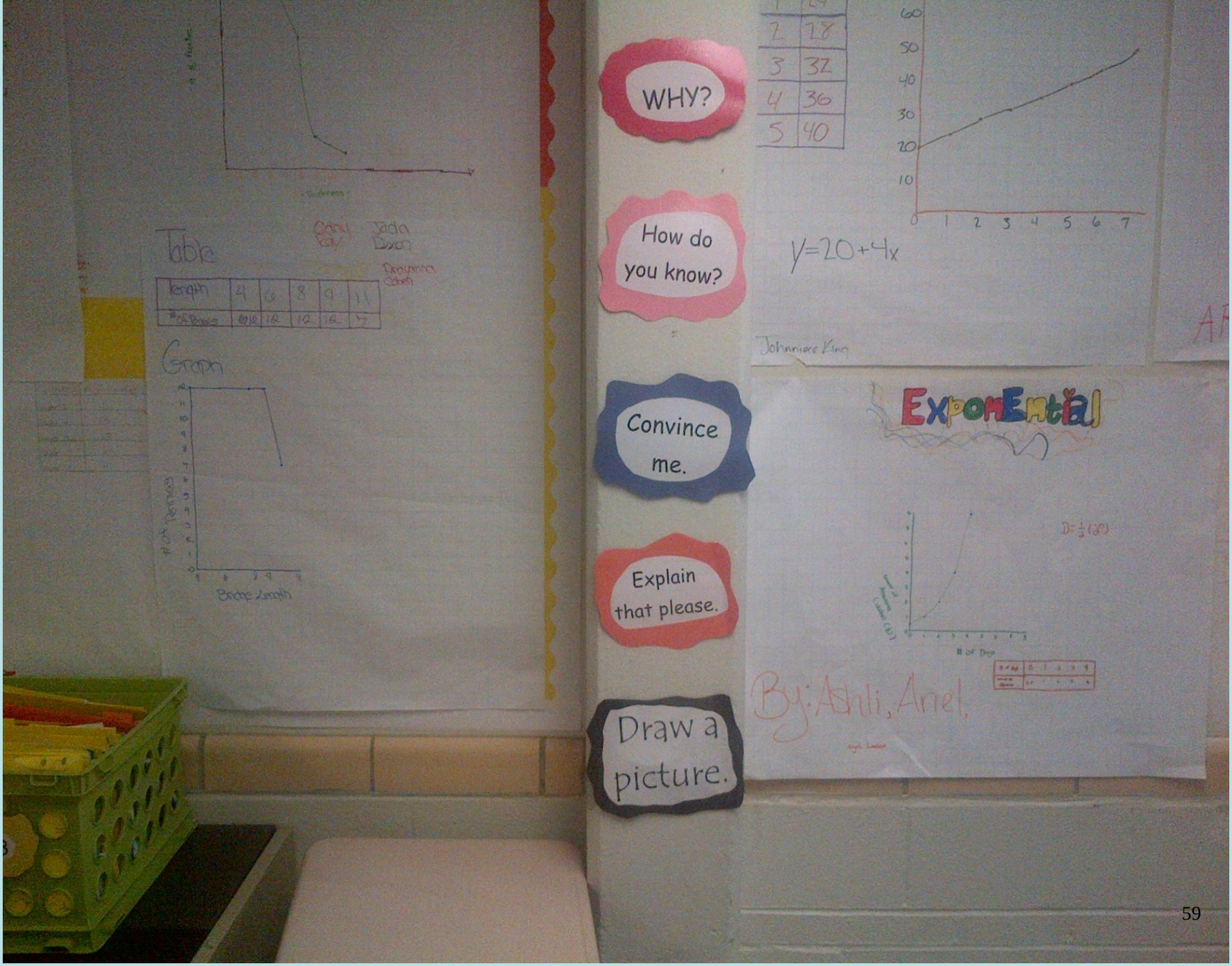
See also: "Accessible Math" by Heinemann


WHY?
How do you know?
Convince me.
Explain that please.
Draw a picture.
Table
Graph
Bridge Length
# of Pennies
y=20+4x
EXPONENTIAL
By: Ashli, Ariel

Summary Thoughts

A Progression of Insights

- **We are charged with making math work for a much greater proportion of students.**
- **But typical instructional practice of showing, telling and practicing to get "right answers" only works for about 1/3 of our students.**
- **To complicate matters, today's world requires reasoning, solving problems, constructing viable arguments (SMPs).**
- **Thus math classes must reflect a different set of instructional practices – productive struggle, alternative approaches and multiple representations, discourse, explanations, conjectures and justifications (MTPs).**
- **But, this is different, difficult to do, requires time and risk-taking.**
- **Which is why we must have high-quality aligned assessments, collaborative structures and coaching to support envisioning, practicing and providing feedback as we raise quality and impact.**

The four key elements of an effective lesson:

1. **The Math: leaning goals, appropriateness, the big ideas, connections, common errors and misconceptions.**
2. **The Tasks: that is the tasks, problems, activities and their richness, alignment with the goals, their appropriateness, their sequencing.**
3. **The Questions and Discourse: how the tasks are orchestrated and conveyed: directions, grouping, who is doing the talking, scaffolding, reviewing and debriefing.**
4. **The Assessment: the evidence that is gathered to determine how well the learning goals were met.**

Developing 6 to 8 slide lesson guides

- **Objective or Lesson Goal slides – student friendly with underlining to focus on understanding of vocabulary and goal**
- **Warm-up – to get the blood flowing and focus on key prerequisite understandings**
- **Tasks with Questions and all necessary graphics – so others can use without additional support**
- **WDYLT**
- **Exit Slip**

What we know
(but too often fail to act on)

People won't do what they can't envision,

People can't do what they don't understand,

People can't do well what isn't practiced,

But practice without feedback results in little change, and

Work without collaboration is not sustaining.

Ergo: Our job, as a professional, at its core, is to help people envision, understand, practice, receive feedback and collaborate.

Thank you.

Now go forth and start shifting YOUR school culture toward greater collegial interaction and collective growth that results in better instruction and even higher levels of student achievement.