Leveraging Arithmetic to Build Infrastructure for Algebraic Success

NCTM INNOV8 CONFERENCE 2016

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Puzzle: Who Am I?

Who Am I?

- The product of my digits is 7.
- The sum of my digits is 8.
- My units digit is greater than my tens digit.

t	u

Puzzle: Who Am I?

Who Am I?

- t is a perfect square.
- $h - u = 0$
- The sum of h and u is 13 more than t.
- I am divisible by 3.

h	t	u

Let's Go Meta

Think about the "Who am I?" Puzzle.

- What did you **do**?
- What were you **thinking about**?
- What **questions** were you asking yourself?

- What **mental habits** do you feel you applied? Discuss with your table-mates.

A Few Ground Rules

- I'll share these slides — please put your email on the list going around, or at **http://v.ht/innov8**
- Please Tweet! Tag **#NCTMInnov8** or **@revuluri**
- The more we **talk**, the more our brains grow — please talk to neighbors, and pause me if needed
- Learning involves disequilibrium. I'll try to stretch or challenge your thinking — please **reciprocate**!
- **Do what you need** to so you can do what your students need you to do — and don't be shy!

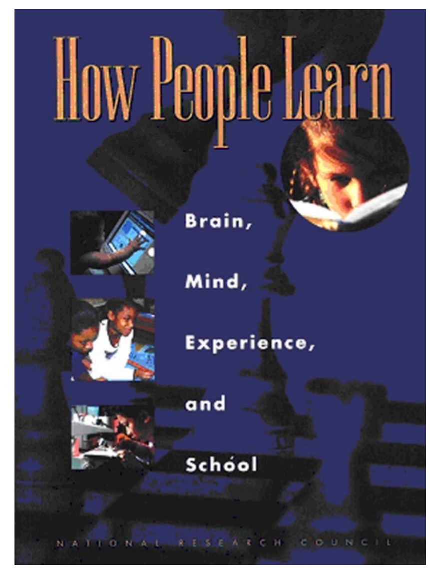
Students’ minds

Misconceptions

So what do we do?

“Teachers must draw out and work with the pre-existing understandings that their students bring with them.”

“mere teaching, no matter how precise, is insufficient to overcome widespread naïve and erroneous thinking about key ideas”



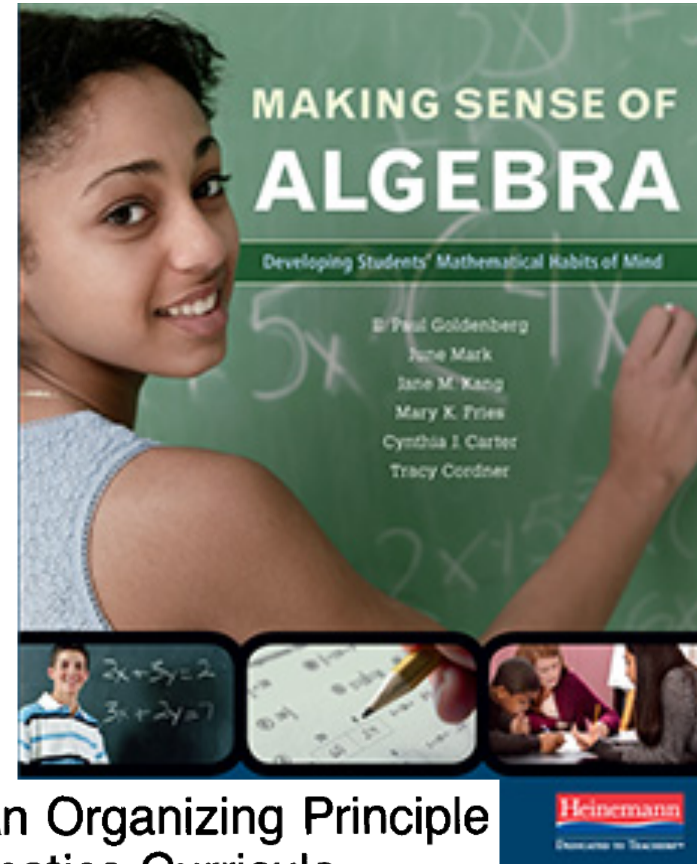
Use prior knowledge as raw material

Goals for Today

- Experience a few activities
- Share a few big, useful ideas
- Expose to more activity resources
- Build community here, and beyond
- Provoke ongoing action and sharing

Habits of Mind

"A habit of mind, then, is a **way of thinking** – almost a **way of *seeing*** a particular situation – that comes so readily to mind that one does not have to rummage in the mental toolbox to find it."

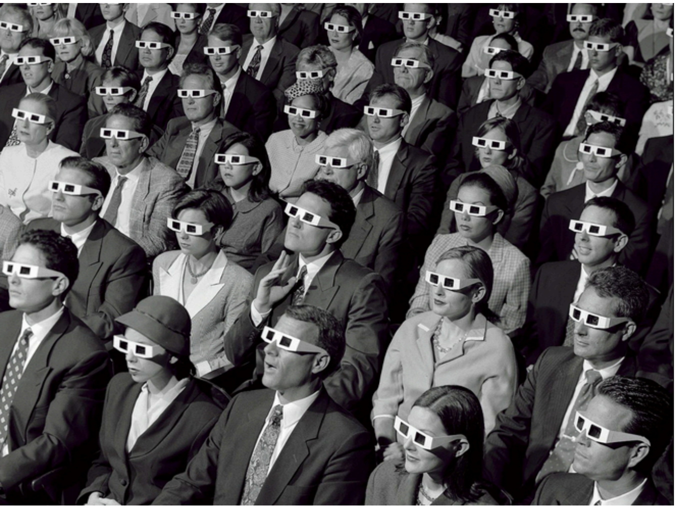


Habits of Mind: An Organizing Principle for Mathematics Curricula

Al Cuoco
E. Paul Goldenberg
June Mark
Education Department Center, Inc., Newton, MA

Mathematically Proficient Students

"they cannot be spectators, simply using and perhaps appreciating the results of mathematicians' work; they must be mathematicians (at their own level, of course), **doing and making mathematics themselves**."



Mathematical habits of mind

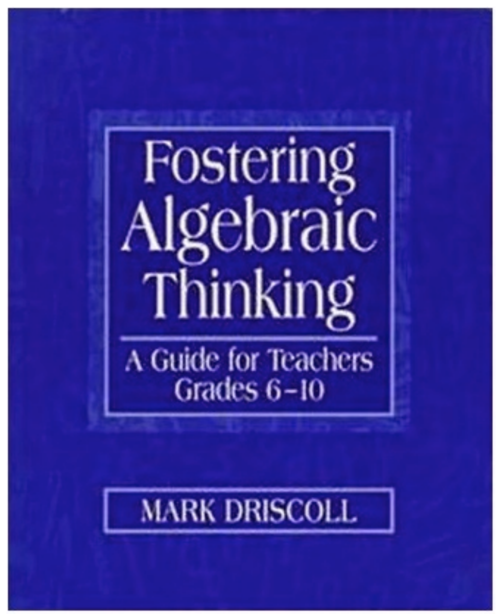
- An I-can-puzzle-it-out disposition
- Characterize problems, solutions in precise ways
- Subdivide and explore problems
(perhaps by posing new, related problems)
- Tinker with problems
(concretely, or with thought experiments)
- Seek, articulate, and use underlying structure
- Choose approaches both strategically and flexibly

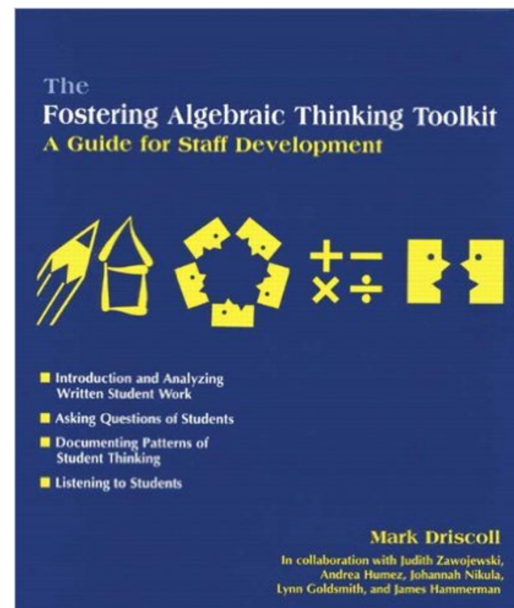
Making Sense of Algebra: Developing Students' Mathematical Habits of Mind (Goldenberg, *et al.*, 2015)

There are Also Domain-Specific Habits

Algebraic Habits of Mind (Driscoll)

- Doing and Undoing
- Building Rules to Represent Functions
- Abstracting from Computation

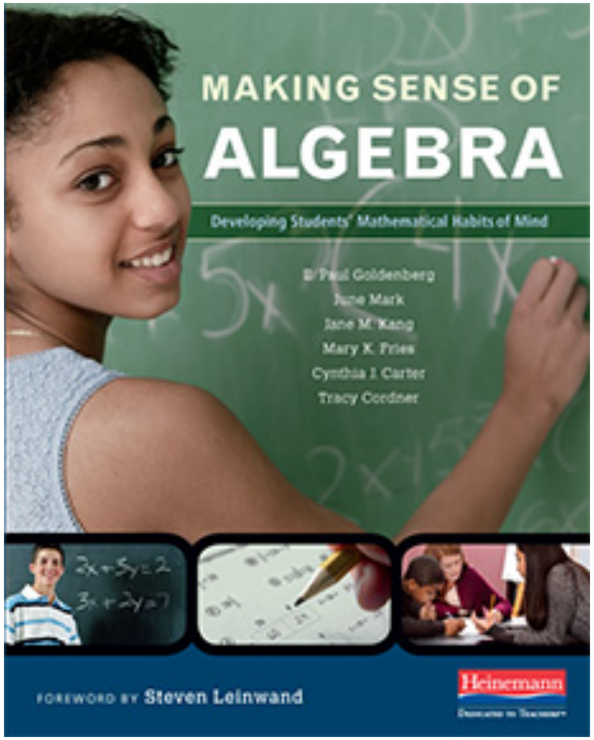




Fostering Algebraic Thinking: A Guide for Teachers Grades 6–10 by Mark Driscoll (1999)

Algebraic Habits of Mind

- Describing Repeated Reasoning
- Puzzling and Persevering
- Seeking and Using Structure
- Using Tools Strategically
- Communicating with Precision

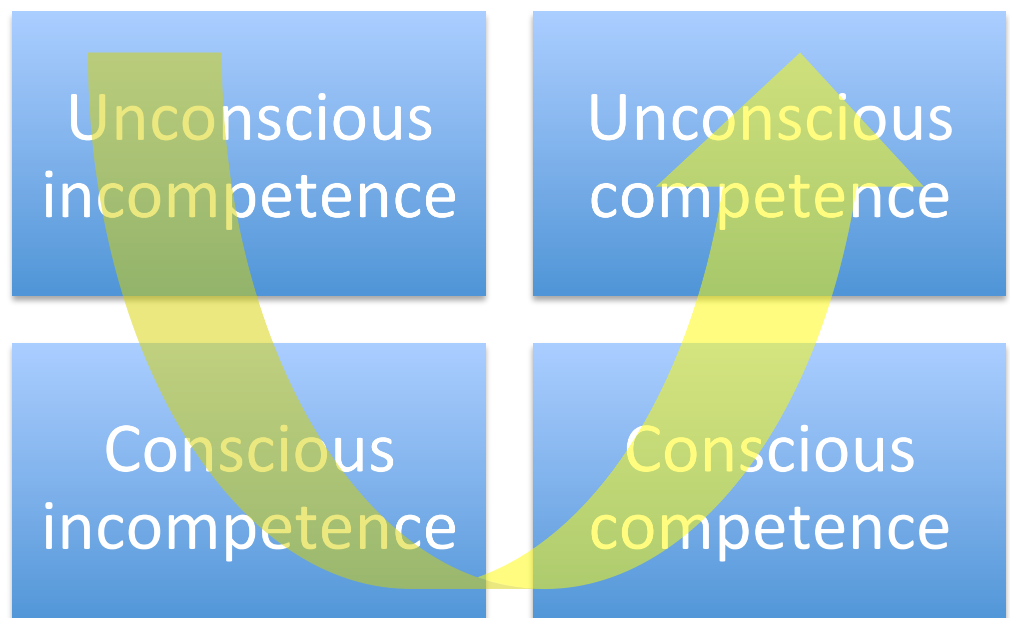


- How can we get students to engage in these habits while working in arithmetic contexts?
- Be aware of your use of these habits today…

How do these behaviors become true habits — second nature?

"Acquiring a habit may require a fair amount of **experience** doing things self-consciously, just as learning to drive is self-conscious before it becomes a habit and fluid and natural."

(Maslow's Four Stages of Learning)



Algebraic Habits Create Infrastructure

Algebraic habits "build mathematical infrastructure; they **focus attention** on algebraic properties and structure, and they **reduce the amount of attention** students will need to pay to raw calculations."

Algebra provides the Logic

Some assume "that knowing one way of doing each kind of problem – a fixed, simplified set of rules — is easier to learn than some more lofty and amorphous "'way of thinking" or a set of alternative strategies. Research, not philosophy, should ultimately dictate whether or not this is the best approach, but it certainly is not an obvious open-and-shut case….

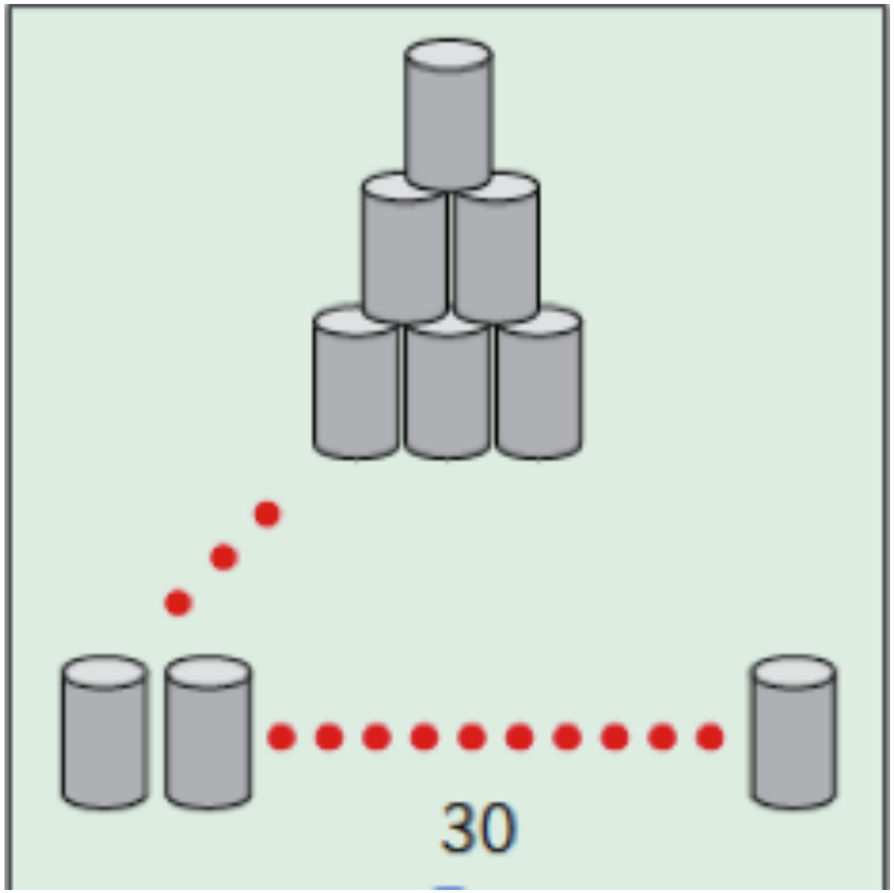
Stuff that you can figure out or think your way through – **stuff that "makes sense" — is generally much easier.**"

(*page 5*)

Making Sense of Algebra: Developing Students' Mathematical Habits of Mind (Goldenberg, *et al.*, 2015)

Arithmetic Gives Students a Place to Put Their Ladder to Success

If there are 30 cans in the bottom row, how many cans are in the whole stack?



31?

10?

Explain your reasoning.

Roy, Safi, and Graul — "Stacking Cans: Abstracting from Computation" (*MTMS* 21:5)

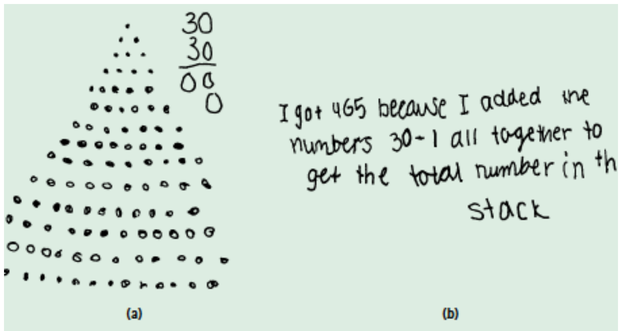
What Methods Might Students Use?

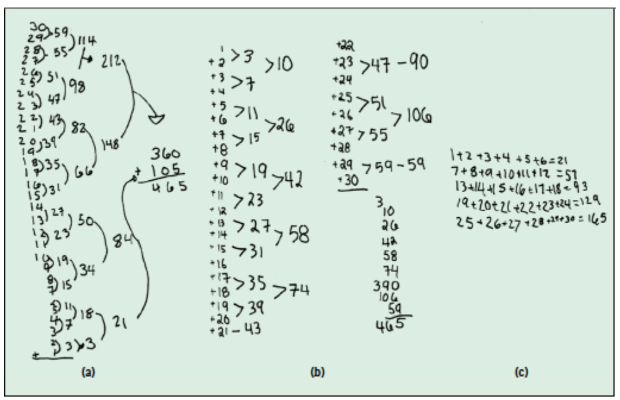
Share your strategy (or strategies) with your neighbors.

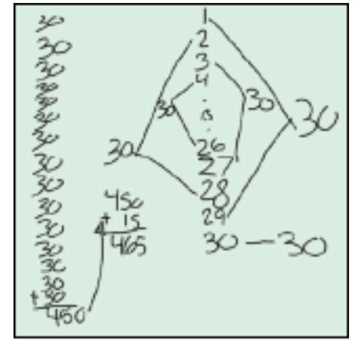
Then, think about other ways students might go about solving this problem.

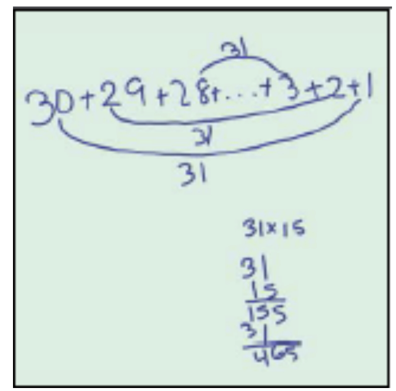
What connections do you see among the methods?

What Methods Might Students Use?

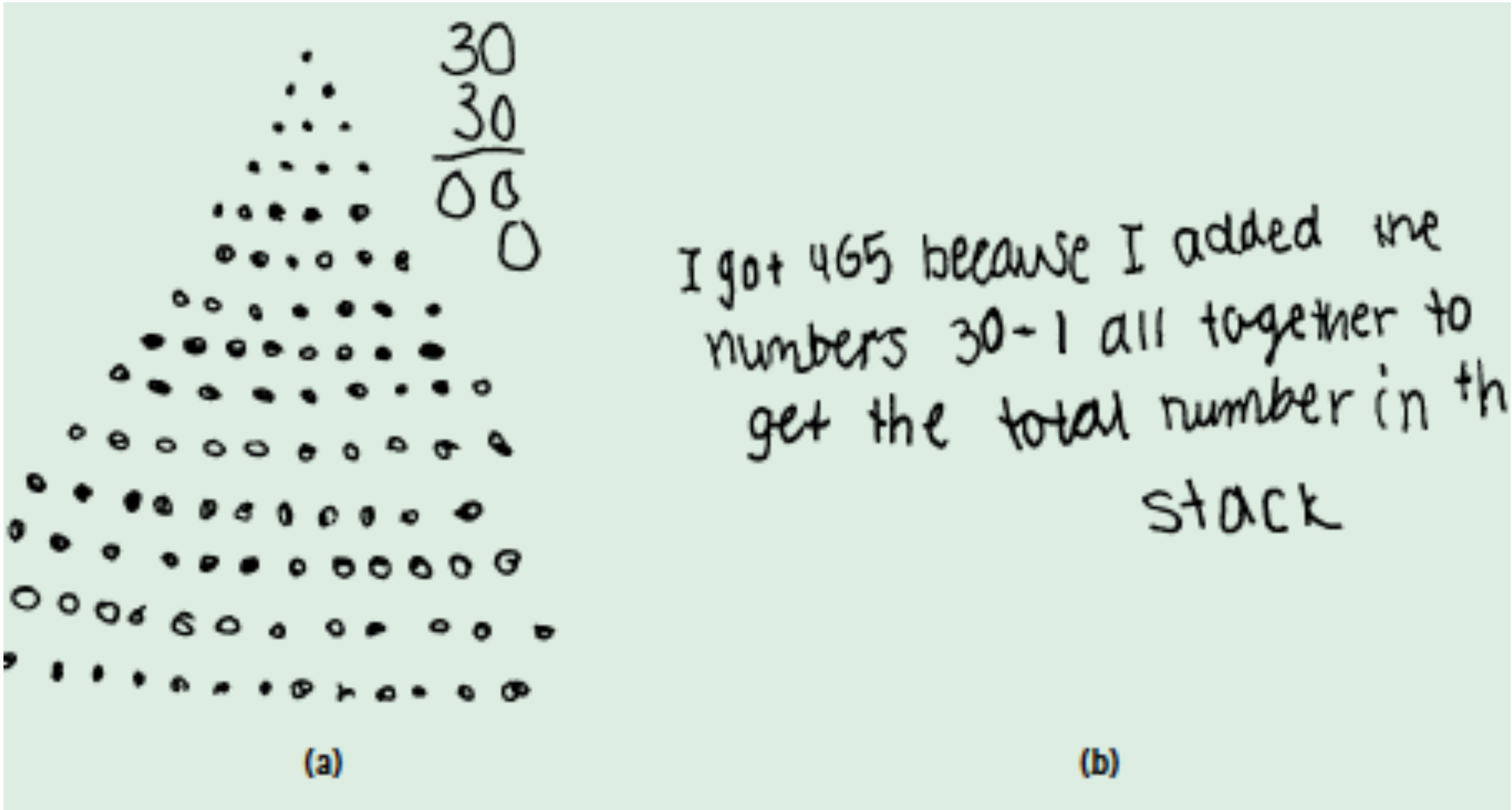




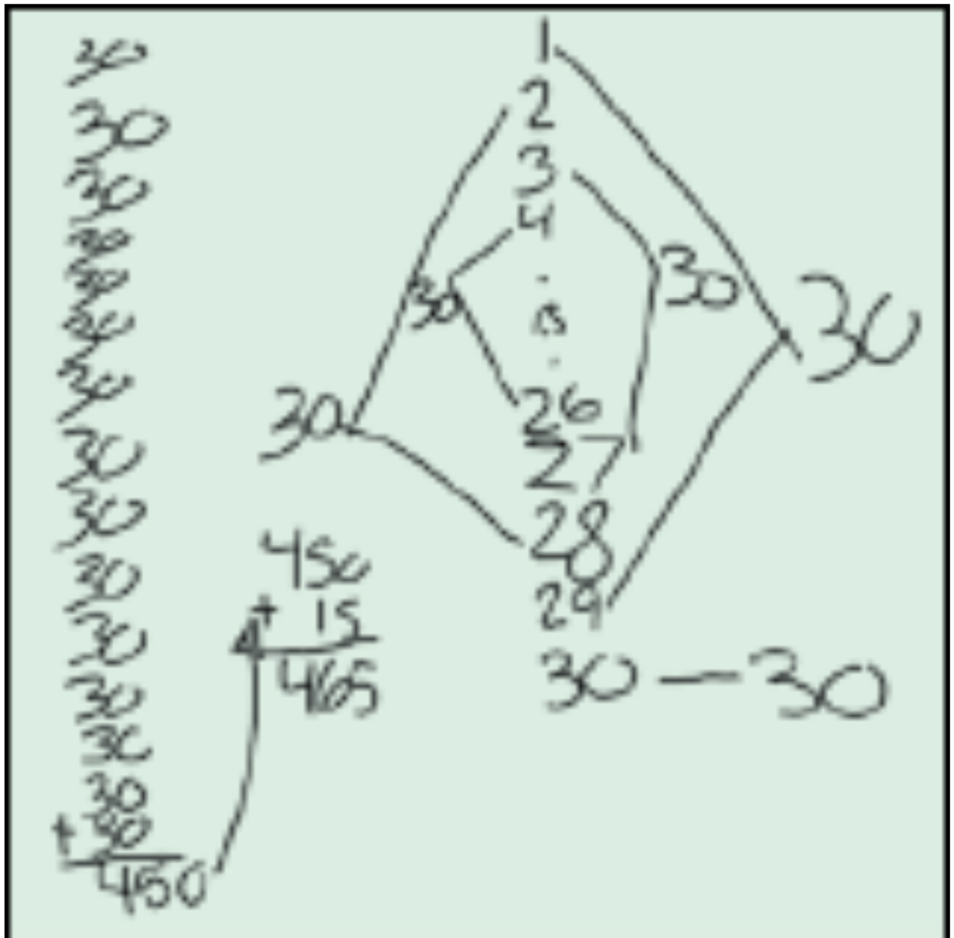


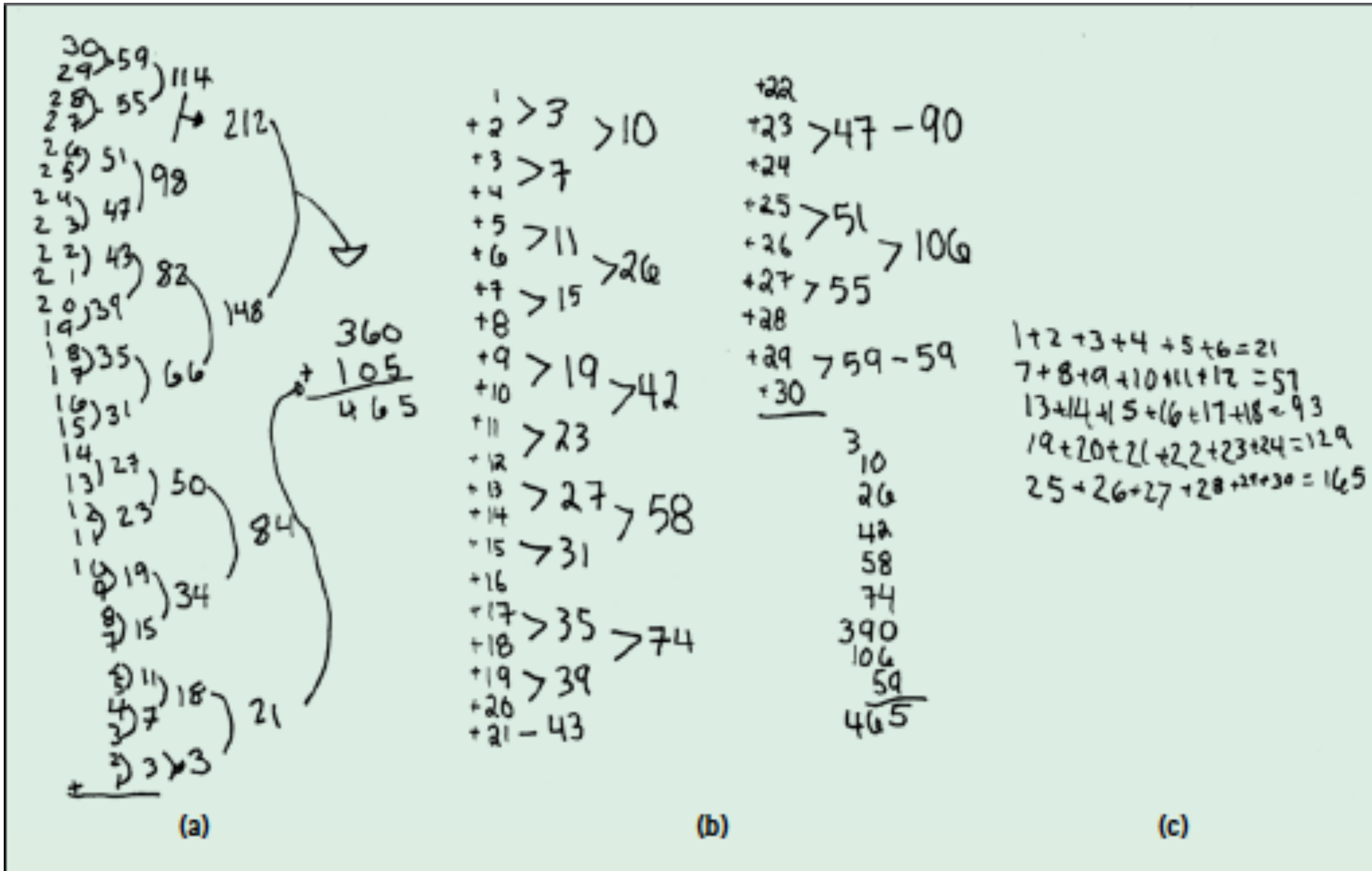


- How did each student solve the problem?
- Compare and contrast two of the sample solutions.
- How might a student generalize from each sample?


30
30
00
0
I got 465 because I added the numbers 30-1 all together to get the total number in th stack

(a) (b)


30
30
30
+ 30
450
456
+ 15
465
1
2
3
4
26
27
28
29
30 — 30


1+2+3+4+5+6=21
7+8+9+10+11+12=57
13+14+15+16+17+18=93
19+20+21+22+23+24=129
25+26+27+28+29+30=165

(a) (b) (c)

31

$$30+29+28+\dots+3+2+1$$

31

31

$$31 \times 15$$

$$\begin{array}{r} 31 \\ 15 \\ \hline 155 \\ 31 \\ \hline 465 \end{array}$$

Let's Go Meta

Think about the Stacking Cans Problem.

- What did you **do**?
- What were you **thinking about**?
- What **questions** were you asking yourself?

- What about this experience promoted the building of algebraic habits of mind?

We Must Liberate Our Struggling Learners from Rote Procedures Alone

- Few struggling students may have had many prior opportunities for sense-making in math
- Through **experiences**, students can still learn properties of real-number operations through their own strategies
- Let's take a quick "tour" of a few lesson structures…

What value of *x* makes this true?

$$7+6=x+5$$

$$43+28=x+42$$

$$28+32=27+x$$

$$67+83=x+82$$

$$12+9=10+8+x$$

$$345+576=342+574+x$$

$$46+28=27+50-x$$

Find the value of each variable

$$3+3+3+3=m\times 3$$

$$5\times n=4+4+4+4+4$$

$$7+8+8+8=7+p\times 8$$

$$8+6+6+6+6=q+5\times 6$$

True or False?

$$9\times 8=10\times 8-8$$

$$4+4+4=3+3+3+3$$

More Relational Thinking

What is the value of each symbol?

2 – 1 = 5

 + ■ = 1

 – 2=★

What is the value of each symbol?

◆ + 2 = 5○

3○ = 12

2○ = ◆ + 2

Find the values of *x* (and *y*)

$$3x = 0$$

$$9x = 0$$

$$0 = -27x$$

$$x - 8 = 0$$

$$x + 1 = 0$$

$$x + 546 = 0$$

$$2(x-9) = 0$$

$$130(x-15) = 0$$

$$6(x+2) = 0$$

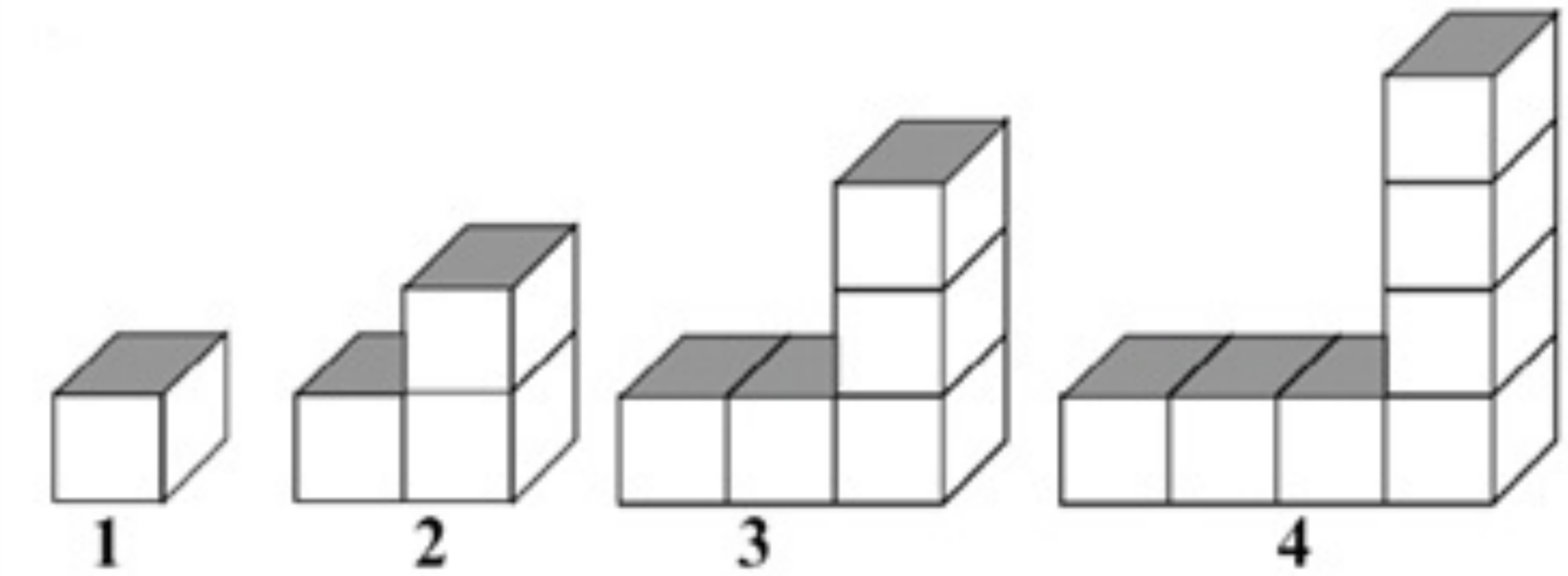
$$xy = 0$$

$$x(x-1) = 0$$

$$(x-4)(x-3) = 0$$

$$(x-7)(x+2) = 0$$

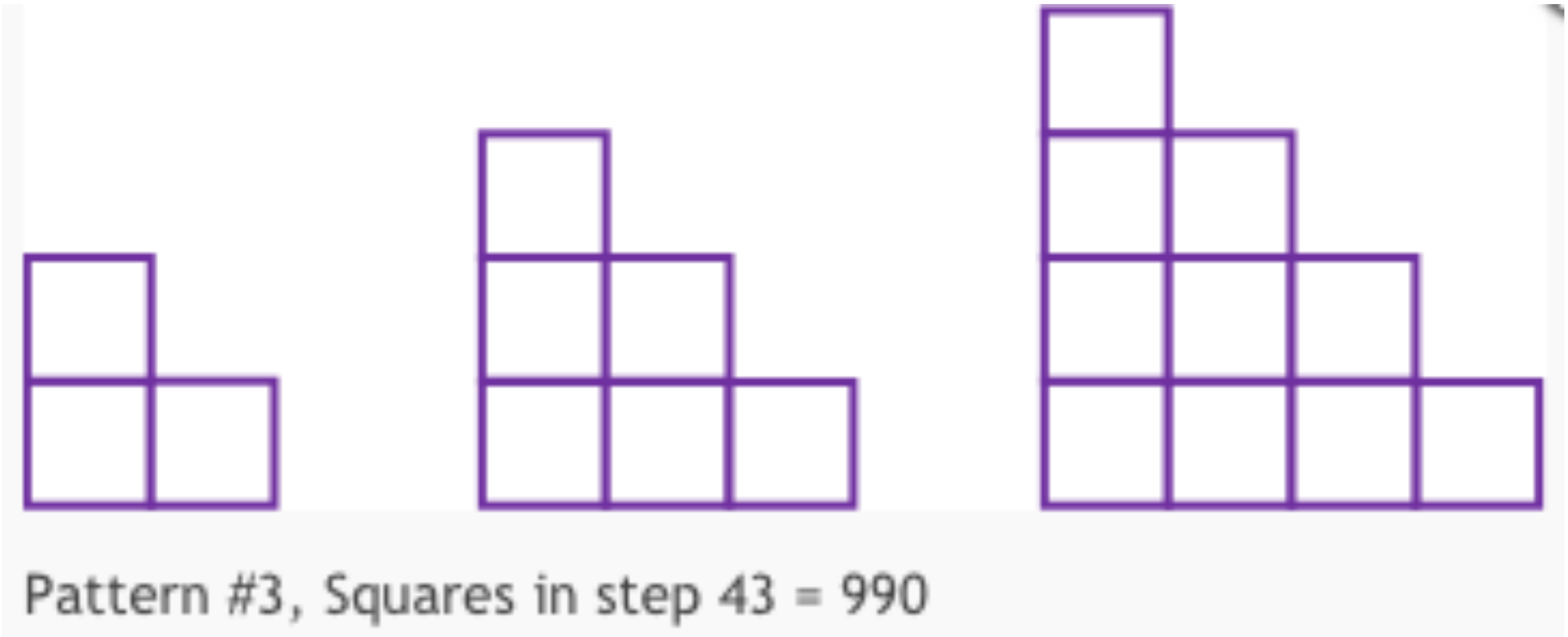
Visual Pattern



Pattern #2, Blocks in step 43 = 85

From Fawn Nguyen's **Visual Patterns** site
(http://www.visualpatterns.org/)

Visual Pattern



Also from Fawn Nguyen's **Visual Patterns** site
(she also runs http://www.mathtalks.net)

Math Talk Resources

- **Contact me to learn about my math talks sessions**
- **Examples**: SD Secondary Number Sense Routines
- ***Number Talks****: Helping Children Build Mental Math and Computation Strategies, Grades K-5* (Parrish)
- ***Making Number Talks Matter*** (Humphreys & Parker)
- Great articles: **Mental Mathematics beyond the Middle School** (*MT*), "**Never Say Anything a Kid Can Say**" and "**Orchestrating Discussions**" (*MTMS*)
- More **resources** online — Math Perspectives, Math Solutions sites (search for Ruth Parker, Cathy Young)

Leveraging Dynamic Algebra

- **Visually juxtaposing multiple methods** to the same problem is a powerful way to support students connection-making and generalizing
- **Dynamic algebra tools** are another way to help — they support strategies like "guess, check, generalize" or take them to a next level
- Good initial activities (**student.desmos.com**): Central Park (**68m4**), Marble Slides, and more

Puzzles Can Build Habits of Mind

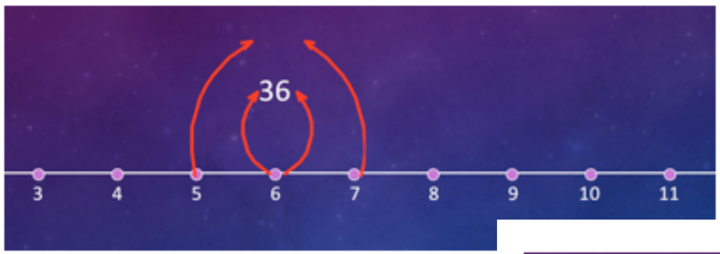
- **Puzzling activities** can support students' practice of SMP (especially looking for, expressing, and using structure, regularity in repeated reasoning)

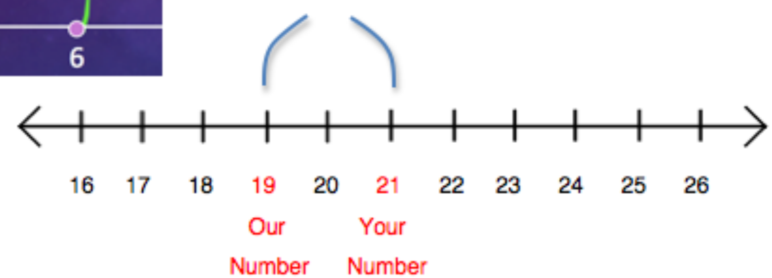
- Puzzles can be low-tech or high-tech; either way, they often yield the most mathematical learning when students work together and talk about it

- Same EDC team (as *MSA* and "Who Am I?") has mobiles, Mystery Number, Mystery Grid online (solveme.edc.org) & on paper (ttalgebra.edc.org)

Productively Perplexing Lessons

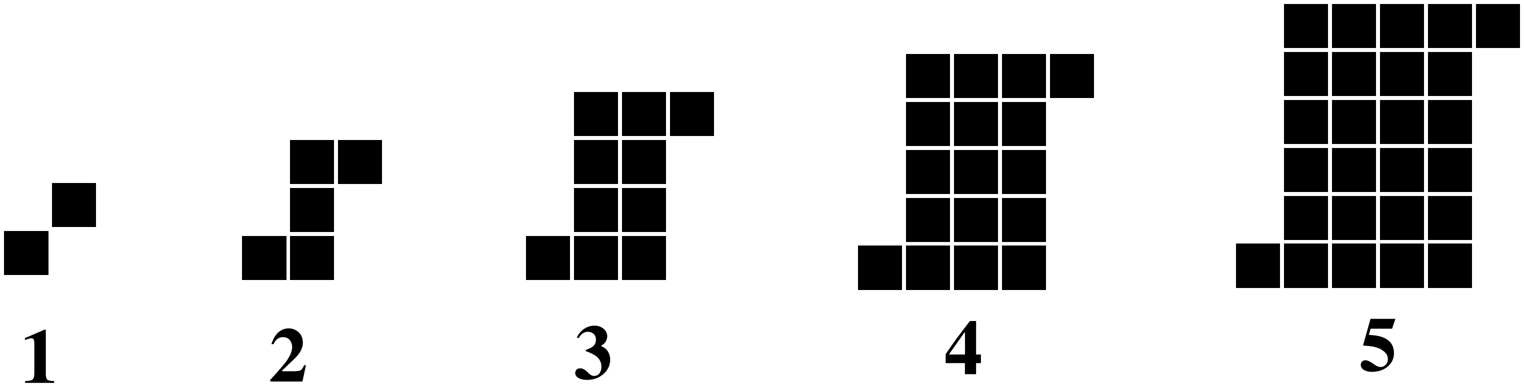
Example: Silent Number Line activity





From **Michael Wiernicki's NCTM blog**
[*http://www.nctm.org/Publications/Mathematics-Teaching-in-Middle-School/Blog/Multiplication-Fluency-in-Middle-School_-Part-3*](http://www.nctm.org/Publications/Mathematics-Teaching-in-Middle-School/Blog/Multiplication-Fluency-in-Middle-School_-Part-3)

The S-Pattern Task

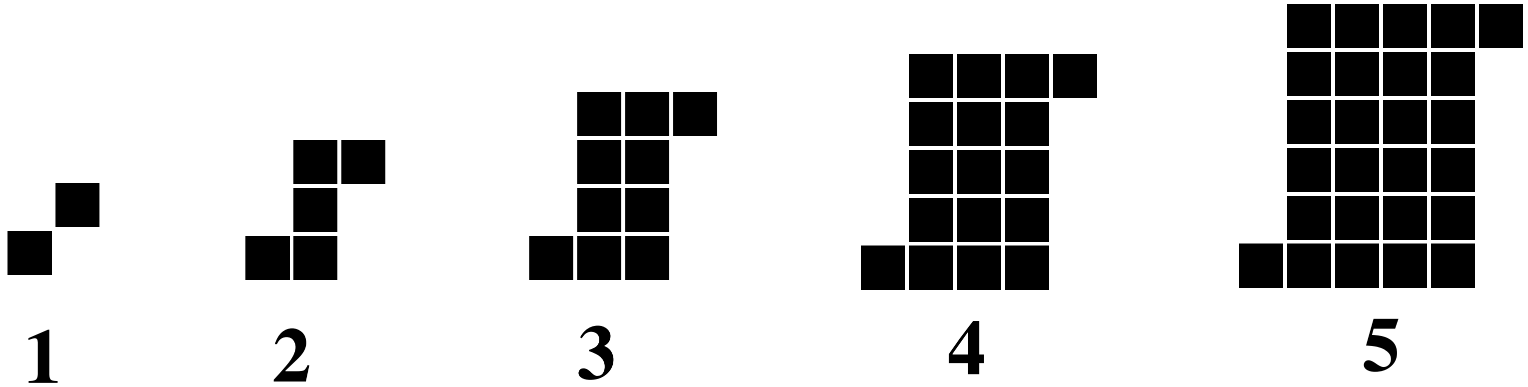


1. What patterns do you notice in the set of figures?

2. How do the figures change from one pattern to the next?

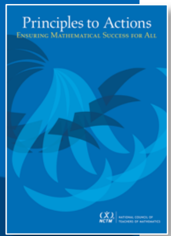
Task adapted from *Visual Mathematics Course II, Lessons 1–10* (The Math Learning Center, Salem, OR)

The S-Pattern Task



1. What patterns do you notice in the set of figures?
2. Sketch the next two figures in the sequence.
3. Describe a figure in the sequence that is larger than the 20th figure without drawing it.
4. Determine an **equation** for the total number of tiles in *any* figure in the sequence. Explain your equation and **show how it relates to the visual diagram of the figures.**
5. If you knew that a figure had 9802 tiles in it, how could you determine the figure number? Explain.
6. Is there a linear relationship between the figure number and the total number of tiles? Why or why not?

Task adapted from *Visual Mathematics Course II, Lessons 1–10* (The Math Learning Center, Salem, OR)



NCTM Principles to Actions Toolkit
S-Pattern Case and Task Resources

Context
Classroom videos (with transcripts and viewing guides)

Task sheets
Templates for Analysis and Teacher and Student Actions
Lesson Guide
Effective Mathematics Teaching Practices

Slides, addressing:

- Mathematics Learning Goals
- Connections to Content and Practice Standards
- Lenses for viewing
- Discussion guides

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NCTM NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Summary

- Algebraic habits of mind provide students with an **infrastructure** to build on their understanding of arithmetic and succeed
- They make the most progress when they get experience by **practicing** them routinely
- Daily activities, rich tasks, puzzles, and dynamic algebra, and Mathematics Teaching Practices are all **resources** to make it happen

Planning to Build Habits

In your solo and team planning, think about:

- Where are **chances** for students to build habits?
- Are **all** students getting these opportunities?
- How do we **build** on prior arithmetic knowledge?

For each activity you consider, think about:

- What will it **feel like** to explore it?
- How could it **support algebraic thinking**?
- What could **make it effective** in class?

Collecting your thoughts

- These structures can help us support students as they build habits and make connections
- We get better through practice **and reflection**
- Please take a moment to gather your thoughts and decide some **next steps** you will take

Thank you!

If you'd like electronic versions of these slides and today's print materials, or you have any questions or suggestions, please put your name on the list and/or contact me at:

sendhil@gmail.com

@revuluri on Twitter

Thank you for joining us today!