MOVING BEYOND MILE-WIDE, INCH-DEEP LEARNING WITH STRUGGLING LEARNERS

Barbara J. Dougherty

November 2016

Innov8

Dougherty, B. J. (2016, November). *Moving beyond mile-wide, inch-deep learning with struggling learners*. Keynote at the National Council of Teachers of Mathematics, Innov8 Conference, St. Louis, MO.

FOCUS ON MISCONCEPTIONS AND HOW TO DIMINISH THEM TO IMPROVE RETENTION AND UNDERSTANDING

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RTI TIER 2 CLASS CAVEATS

- Not tutoring or reteaching that day's lesson (double dosing)
- Focused, targeted, and sequenced instruction (about 30 minutes per day)
- Can be used in Tier 1 (which might lessen the number of students who need Tier 2)

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ALGEBRA SCREENING AND PROGRESS MONITORING (ASPM)

- IES Goal 5, #R324A110262
- Anne Foegen (Iowa State University) and Barb Dougherty (Bill DeLeeuw, Research Assistant)
- Sites in multiple states (IA, MO, KS, MS)
- General and special education high school algebra teachers and students
- Focused on creating measures to determine student progress in developing understanding of algebraic concepts

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PROJECT AIM: ALGEBRA-READINESS INTERVENTION MATERIALS

- IES Goal 2, Development and Innovation, #R324A120364
- Diane P. Bryant (University of Texas-Austin), Barbara Dougherty, Brian Bryant
- Development of intervention modules for use with middle school students to prepare them for algebra

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LESSON LEARNED

How we introduce a new concept/skill is most likely the way that students remember it

It is difficult to 'unteach' something, once students have internalized it in a particular way.

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SHORT-CUTS AND TRICKS ARE POWERFUL!

- In the short-term, they seem like a good idea.
- In the long-term, not so much.

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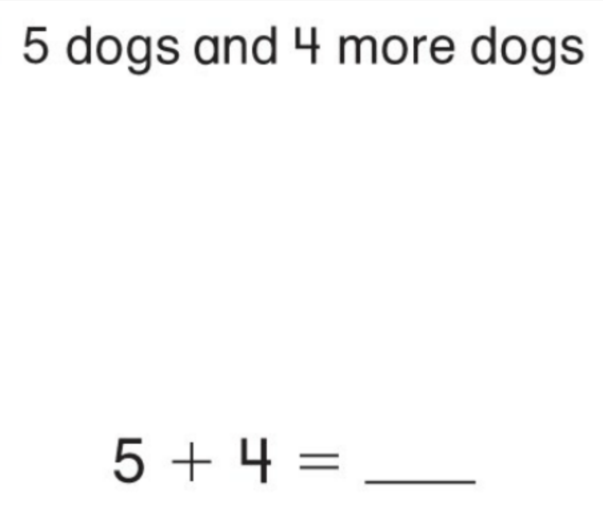
AND SOMETIMES, MISCONCEPTIONS HAPPEN BECAUSE OF THE WAY PROBLEMS ARE PRESENTED



- When problems are presented in a consistent way, students, especially struggling ones, make generalizations.

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WHAT DO WE HAVE TO THINK ABOUT?

- What we teach
  - The mathematics is important
  - What we stress, what we don't stress
  - The types of tasks we give
- How we teach it

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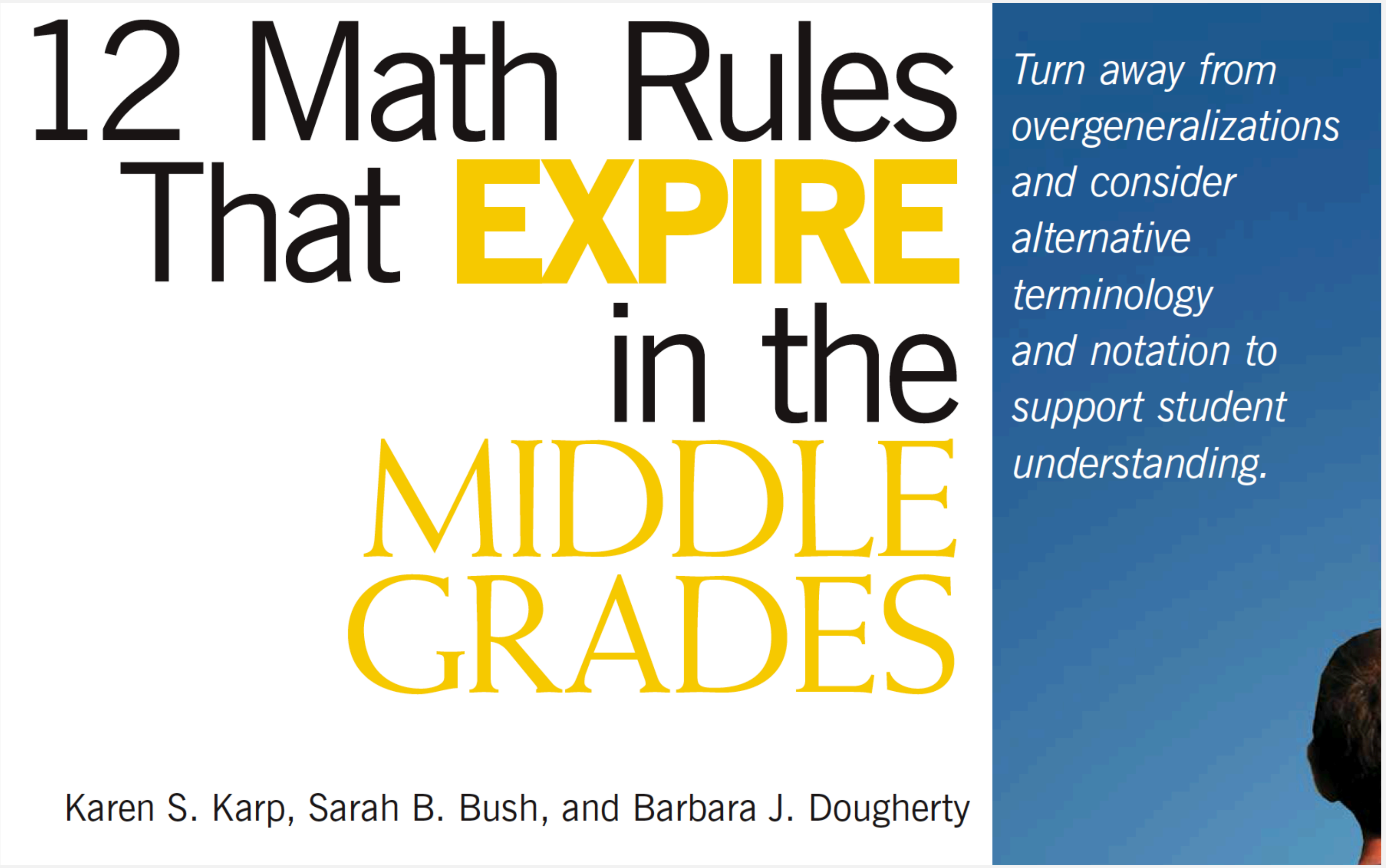
REFERENCE

13 RULES *That Expire*

Overgeneralizing commonly accepted strategies, using imprecise vocabulary, and relying on tips and tricks that do not promote conceptual mathematical understanding can lead to misunderstanding later in students' math careers.

By Karen S. Karp, Sarah B. Bush, and Barbara J. Dougherty



12 Math Rules That EXPIRE in the MIDDLE GRADES

Turn away from overgeneralizations and consider alternative terminology and notation to support student understanding.

Karen S. Karp, Sarah B. Bush, and Barbara J. Dougherty

MISCONCEPTION 1

Dan challenged Amy to write an equation that has a solution of 3. Which equation could Amy have written?

A. $4 - x = 10 - 3x$

B. $3 + x = -(x + 3)$

C. $-2x = 6$

D. $x + 2 = 3$

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MISCONCEPTION 1

Dan challenged Amy to write an equation that has a solution of 3. Which equation could Amy have written?

A. $4 - x = 10 - 3x$	(119/490; 24.3%)
B. $3 + x = -(x + 3)$	(135/490; 27.6%)
C. $-2x = 6$	(95/490; 19.4%)
D. $x + 2 = 3$	(141/490; 28.8%)

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MISCONCEPTION 1

- The answer comes after the equal sign.

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MISCONCEPTION 1

- The 'answer' follows the equal sign.
  - Misunderstanding of what the equal sign represents
  - Misunderstanding of what a solution to an equation is

That the equal sign is a 'do something signal' is a thread which seems to run through the interpretation of equality sentences throughout elementary school, high school, and even college. Early elementary school children … view the equal sign as a symbol which separates a problem and its answer. (Kieran 1981, p. 324)

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EQUAL SIGN–TWO LEVELS OF UNDERSTANDING

Operational: Students see the equal sign as signaling something they must "do" with the numbers such as "give me the answer."

Relational: students see the equal sign as indicating two quantities are equivalent, they represent the same amount. More advanced relational thinking will lead to students generalizing rather than actually computing the individual amounts. They see the equal sign as relating to "greater than," "less than," and "not equal to."

Knuth, E. et. al (2008). The importance of equal sign understanding in the middle grades. *Mathematics Teaching in the Middle School*, *13*, 514–519.

WHY IS UNDERSTANDING THE EQUAL SIGN IMPORTANT?

Table 1 Percent of students at each grade level who provided each type of equal sign definition as their best definition ($n = 375$)

Best Definition	Grade 6	Grade 7	Grade 8
Relational	29	36	46
Operational	58	52	45
Other	7	9	8
No response/ don't know	6	3	1

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WHY IS UNDERSTANDING THE EQUAL SIGN IMPORTANT?

Students who do not understand the equal sign have difficulty in algebra with equations like:

$$3x - 4 = 7x + 8$$

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MISCONCEPTION 1

Given the task:

$$8 + 4 = \square + 5$$

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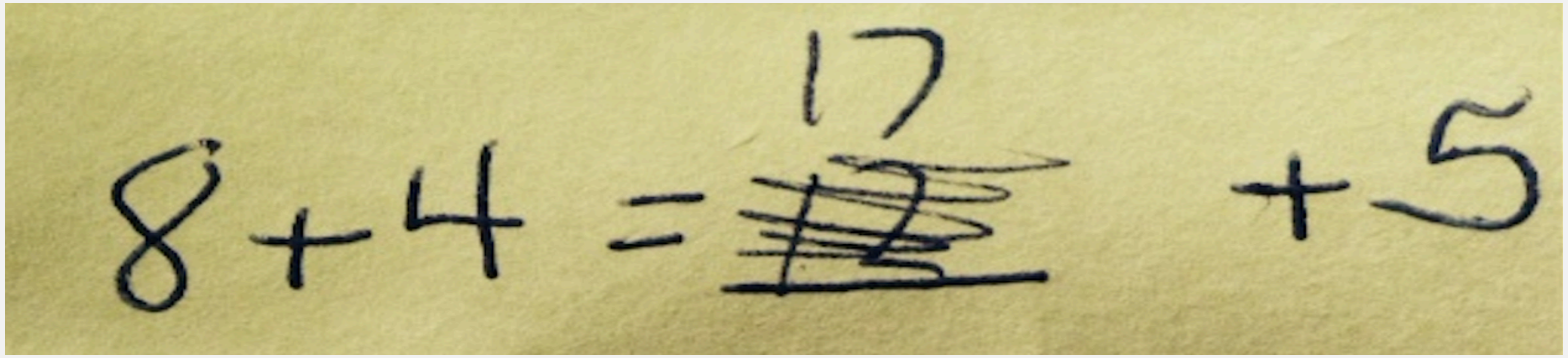
MISCONCEPTION 1: HUDSON'S WORK



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HUDSON'S 2ND GRADE EXPLANATION

Hudson: Well, it's really kind of easy 'cause the answer comes after the equal sign, then you add the 5 later. This is easy math 'cause when you have numbers you just do whatever to them. Like add, subtract. And I think next year we learn how to multiply.

Nana: How do you know if your answer is correct?

Hudson (looking like why are you asking me that?): Oh, it's like every math thing. Your teacher tells you. As long as you show your work, she won't get mad.

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How did other students perform on the same problem?

Table 1 Students in various grades had different responses to make the sentence $8 + 4 = \square + 5$ true.

Responses	Grades		
	1 and 2	3 and 4	5 and 6
7	5%	9%	2%
12	58%	49%	10%
17	13%	25%	21%
12 and 17	8%	10%	2%

Source: Adapted from Falkner, Levi, and Carpenter 1999, p. 132

MISCONCEPTION 1: WATCH 4S!

13 x 10 = 130 + 4 = 134 – 8 = 126

Stringing together expressions/calculations

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MISCONCEPTION 1: STRATEGY 1

Build on what students have done in elementary grades

125 < 152 by 27

152 > 125 by 27

$\frac{3}{4} < \frac{7}{8}$ by $\frac{1}{8}$

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MISCONCEPTION 1: STRATEGY 1

125 < 152 by 27
152 > 125 by 27

→

152 = 125 + 27
152 = 27 + 125
152 – 27 = 125
152 – 125 = 27

$\frac{3}{4} < \frac{7}{8}$ by $\frac{1}{8}$

→

$$\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$$

$$\frac{1}{8} + \frac{3}{4} = \frac{7}{8}$$

$$\frac{1}{8} = \frac{7}{8} - \frac{3}{4}$$

$$\frac{3}{4} = \frac{7}{8} - \frac{1}{8}$$

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MISCONCEPTION 1: STRATEGY 2

Provide a mixture of problems that have the expressions on both sides of the equal sign.

❑= 8.75 – 4.27

4 – (–2) = ❑+ 14

–2.53 + ❑ = 6 + 4.31

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MISCONCEPTION 1: STRATEGY 2

- Use precise language
- Use appropriate and consistent language
- Read = as “is equal to”
  - Be careful about saying—
    - Solve an expression
    - Answer to an equation

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MISCONCEPTION 2

Solve for p:

$$16 - p = 7$$

How do you think your students would solve the equation?

How would you LIKE for them to solve the equation?

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MISCONCEPTION 2

In Year 1 of our project, 1,201 students completed a skill progress monitoring measure in which this was the first item.

67% gave the correct answer.

18% gave an incorrect answer.

15% skipped the item.

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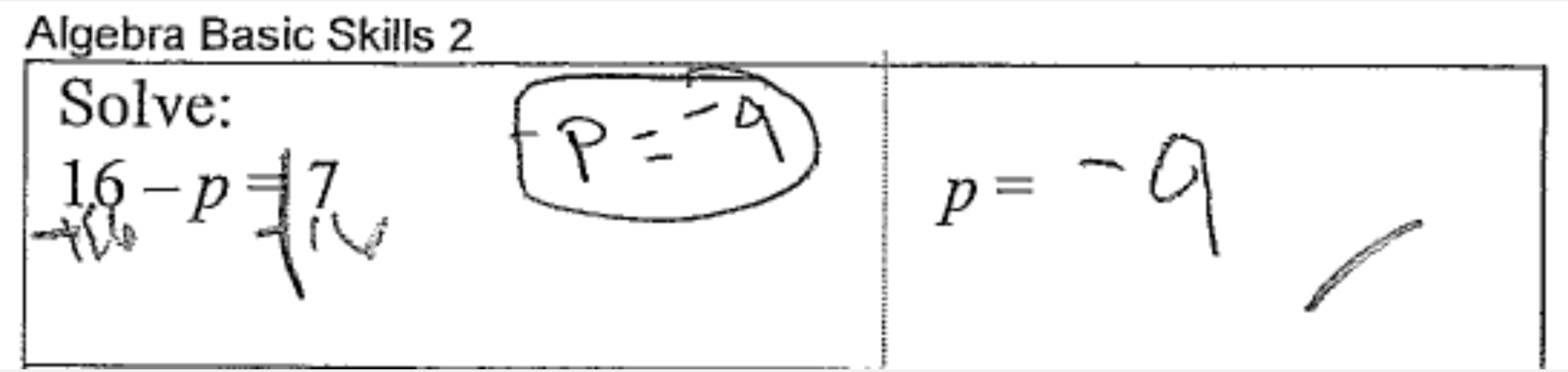
MISCONCEPTION 2



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STUDENT COMMENTS

$16 - \square = 7$

$16 - p = 7$

"These are different. The top one is just arithmetic, like first grade stuff. The second one is real algebra. You have to show your steps when you do algebra."

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MISCONCEPTION 2

You cannot use logical reasoning or intuition because you have to show all of your steps.

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MISCONCEPTION 2: STRATEGY 1

Change Show Your Work to SHOW YOUR THINKING.

Use questions that motivate students to use what they know about one problem to solve another problem.

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MISCONCEPTION 2: STRATEGY 1

If g – 227 = 543, what does g – 230 equal?

How do you think your students would solve this?

How would you LIKE for them to solve it?

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MISCONCEPTION 2: STRATEGY 1

If $g - 227 = 543$, what does $g - 230$ equal?

194 out of 488 students (39.8%) responded 540.

122 out of 488 students (25%) responded 546 OR 770

172 out of 488 students (35.2%) incorrectly responded with other values that included—

–874 37 16,939 1121 1929 –703

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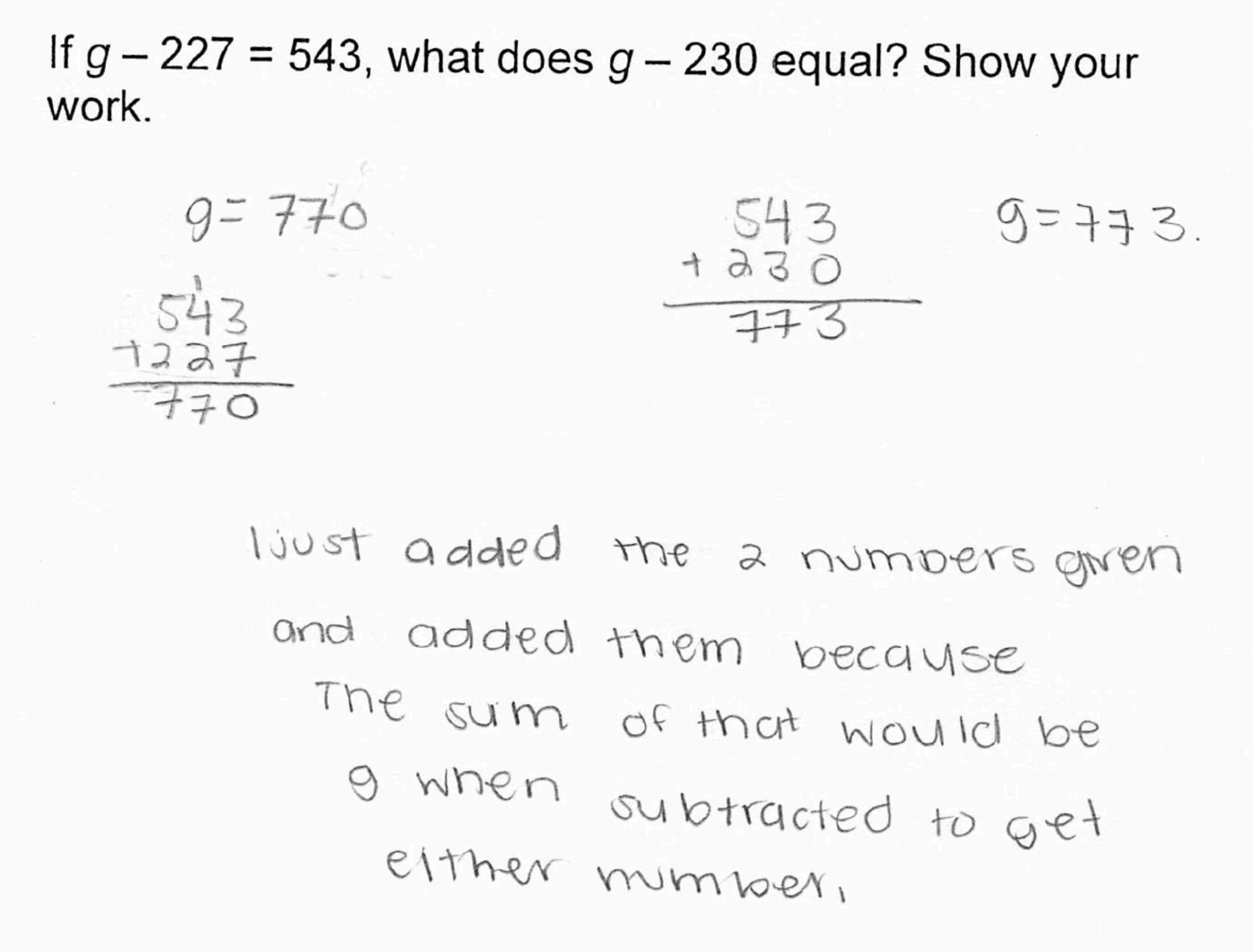
MISCONCEPTION 2: STRATEGY 1



If $g - 227 = 543$, what does $g - 230$ equal? Show your work.

g = 770

  543
+ 227
  770

  543
+ 230
  773

g = 773.

I just added the 2 numbers given and added them because The sum of that would be g when subtracted to get either number.

MISCONCEPTION 2: STRATEGY 1

- Explicitly ask students to use multiple methods
- When multiple methods are presented, compare and contrast them.

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MISCONCEPTION 2:STRATEGY 2

Focus on relationships rather than only procedures.

For example: Solving equations is not about ‘moving’ things from one to the other or doing the opposite.

It’s about understanding relationships.

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MISCONCEPTION 3

Multiplication and addition make bigger.

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ADDITION MAKES BIGGER

- 8 + 3 = 11
- 4 + 5 = 9
- 2 + 1 = 3

It's Correct!!

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ADDITION MAKES BIGGER

- Maybe not:
  - $0 + 4 = 4$

- And later:
  - $-2 + (-5) = -7$

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WHY DOES IT MATTER?

Mari said, "2*t* is always greater than *t* + 2." Do you agree with Mari?

A. Yes, because multiplication always gives you a larger answer than addition.

B. Yes, because *t* is a positive number.

C. No, because multiplication is not the inverse of addition.

D. No, because it is possible that 2*t* can be equal to or less than *t* + 2.

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WHY DOES IT MATTER?

Mari said, "2*t* is always greater than *t* + 2." Do you agree with Mari?

A. Yes, because multiplication always gives you a larger answer than addition. (312/750; 41.6%)

B. Yes, because *t* is a positive number. (64/750; 8.5%)

C. No, because multiplication is not the inverse of addition. (107/750; 14.3%)

D. No, because it is possible that 2*t* can be equal to or less than *t* + 2. (267/750; 35.5%)

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MISCONCEPTION 3: STRATEGY 1

- Provide problems that focus on structure and motivate reasoning with generalized contexts

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MISCONCEPTION 3: STRATEGY 1

Which represents the greater number or quantity?

1. $2x$ or $x + 2$

2. m or $-m$

3. $3x$ or x^3

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MISCONCEPTION 3: STRATEGY 1

$2x$ or $x + 2$

It depends. What if $x = 0$? What if $x < 0$? What if $x = 0.75$? What if $x = 2$?

m or $-m$

It depends. What if $m = 0$? What if $m < 0$? What if $m > 0$?

$3x$ or x^3

It depends. What if $x = 0$? What if $x = 1$? What if $x < 0$?

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MISCONCEPTION 3: STRATEGY 2

Consider each statement below. Then, write <, >, = or cannot tell (CT) in the blank.

If $a < b < c$ and they are whole numbers, then $\frac{a}{c}$ ____ $\frac{b}{c}$.

If $a < b$ and they are whole numbers, then $\frac{a}{b}$ ____ $\frac{b}{b}$.

If $a < b < c$ and they are whole numbers, then $\frac{a}{b}$ ____ $\frac{a}{c}$.

If $a < b < c$ and they are greater than 0 but less than 1, then $\frac{a}{c}$ ____ $\frac{b}{c}$.

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MISCONCEPTION 4

- Students often believe that mathematics is only a series of steps.
  - When skills are broken down into small pieces with very specific rules, it requires students to put the pieces together to form the whole.

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MISCONCEPTION 4

- Here's how you. . . . . . .
- Now you solve these . . . . . .

- I do
- We do
- You do

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EXAMPLE-BASED TEACHING

Example-based teaching requires students who are struggling to make generalizations about the structure of the class of problems.

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REDESIGNING EXPLICIT INSTRUCTION

- It is NOT direct instruction.
  - Direct instruction is the teacher showing students how to do something or giving factual information in a structured sequence.

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REDESIGNING EXPLICIT INSTRUCTION

- Focusing students attention on particular structures or ideas
  - Asking questions so that students ‘see’ the mathematics
  - Providing tasks that allow students to explore the topic
  - Making assumptions obvious

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REDESIGNING EXPLICIT INSTRUCTION

- Teacher introduces a problem that links to previous learning.
- Students work in pairs or small groups to solve.
- Students share their thinking with the class, critiqued by others and teacher.
- Teacher scaffolds tasks based on misconceptions that are evident in thinking.
- Teacher poses questions throughout that focus students on important ideas and generalizations.

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REDESIGNING EXPLICIT INSTRUCTION

- Always try to elicit information from students first.
  - You need to hear their misconceptions or ideas.

- Trust that students can do it.

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CREATE HIGH EXPECTATIONS

- Critical thinking questions should be asked in every class, every day
- Consistency helps students understand the expectations and move toward higher proficiency

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CHANGING OUR QUESTIONING

Solve for x:

a. $2x + 4 = 3x - 8$

Find the difference:

$70 - 23 = ?$

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REVERSIBILITY QUESTIONS

- Find an equation whose solution is 12.
- Find another equation, with variables on both sides of the equal sign, whose solution is 12.

- Find 2 numbers that have a difference of 47.
- Find 2 more numbers that have a difference of 47.

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REVERSIBILITY QUESTIONS

- Promote the ability to think in different ways
- Allow for multiple answers
- Provide accountability for students to respond
- Give answer, students create the problem

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GENERALIZATION QUESTIONS

- Find a linear equation whose solution is a whole number.
- Is it possible to predict if the solution of an equation is a whole number? Why or why not?
- What is the largest number of digits that can be in the difference when you are subtracting 2 two-digit numbers?
- What is the smallest number of digits that can be in the difference when you are subtracting 2 two-digit numbers?

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GENERALIZATION QUESTIONS

- Ask students to find and describe patterns
- Use the patterns to solve problems or create specific examples of a pattern
- Moves student thinking to look at broader aspects rather than specific problems
- What patterns do you notice? What observations can you make?

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FLEXIBILITY QUESTIONS

Solve:

$2x - 8 = 3x + 4$

Solve it another way.

Find the difference:

$70 - 23 = ?$

Find the difference another way.

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FLEXIBILITY QUESTIONS

Solve:

$2x - 8 = 12$

$2(x + 2) - 8 = 12$

$2(2x + 2) - 8 = 12$

$70 - 23 = ?$

$70 - 27 = ?$

$73 - 27 = ?$

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FLEXIBILITY QUESTIONS

- Ask students to solve a problem in multiple ways OR to use what they know about one problem to solve another one
- Provide a means to compare and contrast solution methods
- Focus on relationships
- Solve the problem in another way.
- How are these problems alike? How are they different?

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FINAL THOUGHTS: AVOID RULES OR GENERALIZATIONS THAT EXPIRE

- Key words
  - Add when you see 'altogether'
  - Subtract when you see 'left'
- Multiplication is the opposite of division
- You always put the variable first in an expression ($y + 3$ rather than $3 + y$)
- The variable always goes on the left of the equation when you are solving it.

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FINAL THOUGHTS: EXPLICIT DISCUSSIONS

- Ask questions that focus students on specific features, characteristics, or structure

  Why is the sum of 75 + 21 greater by 2 from the sum of 73 + 21?

  Given the expression $3x - 2$, what is the effect on the value of the expression as x increases by 4?

  Is $5t$ always greater than t? Why or why not?

Dougherty, B. J. (2016, November). *Moving beyond mile-wide, inch-deep learning with struggling learners*. Keynote at the National Council of Teachers of Mathematics, Innov8 Conference, St. Louis, MO.

CRITERIA TO USE IN BUILDING LESSONS

- Introduce every topic with problem solving
- Ensure every lesson includes five forms of communication
  1. Reading
  2. Speaking
  3. Critical listening
  4. Writing
  5. Multiple representations
- Connect new topics with older ones
- Provide students with 8 – 15 days to move a concept to a skill
- Present challenging problems for **all** students

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MORE FINAL THOUGHTS

- Rather than breaking topics (concepts and skills) into small pieces, think more connected.
- Be consistent in asking cognitively demanding questions.
- Think about assumptions that are made in problems—make them more explicit.
- Articulate expectations for students for every task.
- Make every day a day that includes critical thinking and problem solving.

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CHANGE DOING MATH INTO THINKING WITH MATH

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AVAILABLE SURVEY

University of Hawaii, Curriculum Research & Development Group would welcome your input:

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QUESTIONS
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