Principles to Actions

Effective Mathematics Teaching Practices

The Case of Debra Campbell and the Building a New Playground Task

Geometry

This module was developed by Frederick Dillon, Math Instructional Coach, Ideastream. Video courtesy of Hamilton County School District and the Institute for Learning.

These materials are part of the ***Principles to Actions Professional Learning Toolkit: Teaching and Learning*** created by the project team that includes: Margaret Smith (chair), Victoria Bill (co-chair), Melissa Boston, Frederick Dillon, Amy Hillen, DeAnn Huinker, Stephen Miller, Lynn Raith, and Michael Steele.

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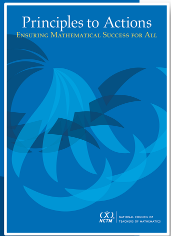

Overview of the Session

- Solve and Discuss two versions of Building a New Playground task

- Watch the video clip and discuss what the teacher does to support her students engagement in and understanding of mathematics

- Discuss the effective mathematics teaching practices of *supporting productive struggle* and *posing purposeful questions.*

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The Tasks

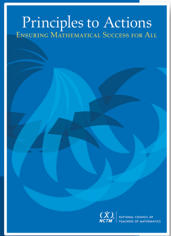
- There are two versions of the Building a New Playground Task -- V1 and V2.
- You will consider both versions of the task and what students could learn from each.
- You will analyze student responses to each version of the task and consider what each student knows and understands.

The Building a New Playground Task V1

The City Planning Commission is considering building a new playground. They would like the playground to be equidistant from the two elementary schools, represented by points A and B. (See Task sheet for diagram.)

Part 1 – Determine at least three possible locations for the park that are equidistant from points A and B. Explain how you know that all three possible locations are equidistant from the elementary schools.

Part 2 – Make a conjecture about the location of all points that are equidistant from A and B. Prove this conjecture.

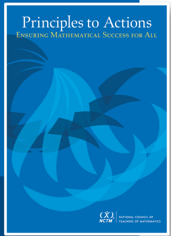


Take Time to Work the Task

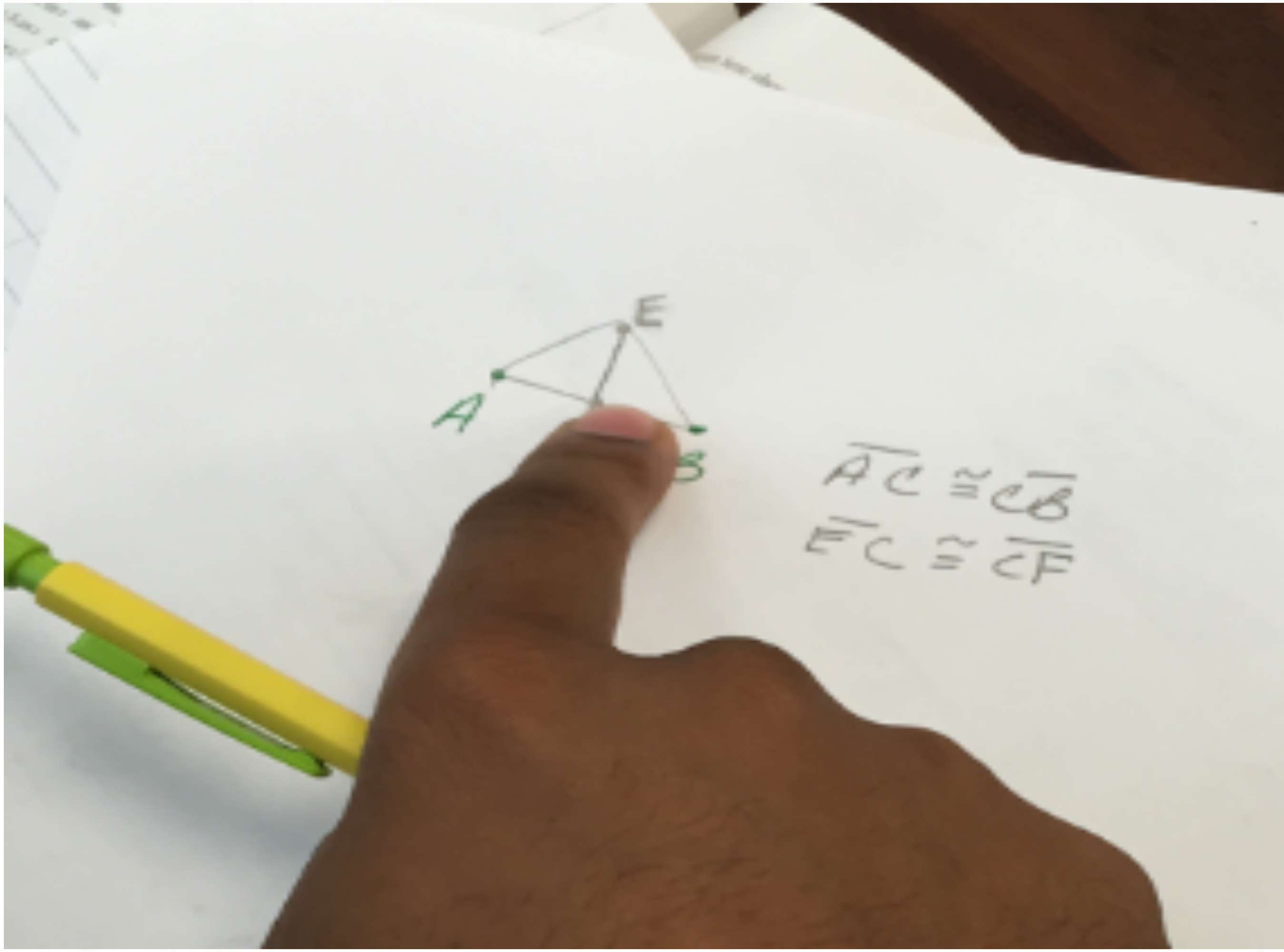
- First, work on the task individually.
- Next, share your solution(s) with your elbow partner.
- Finally, as a small group, discuss your solution methods, think about other possible student approaches to the problem, and consider how you would implement the task.

Analyzing Student Work

- Look at André’s work.
- As you watch André’s solution unfold, make note of what he is doing.



The Building a New Playground Task V1



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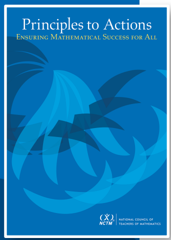

Analyzing Student Work

- What does André know about right angles?
- Would you accept his “construction” of a right angle?
- What can you learn about André’s thinking from his explanation?

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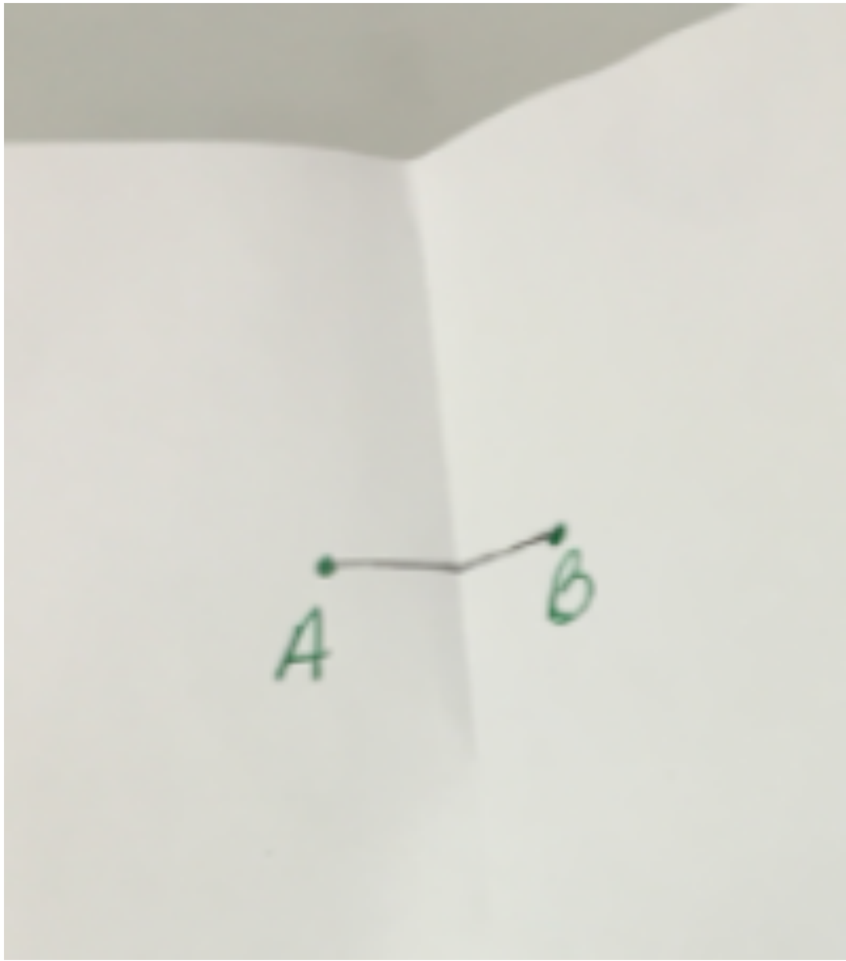
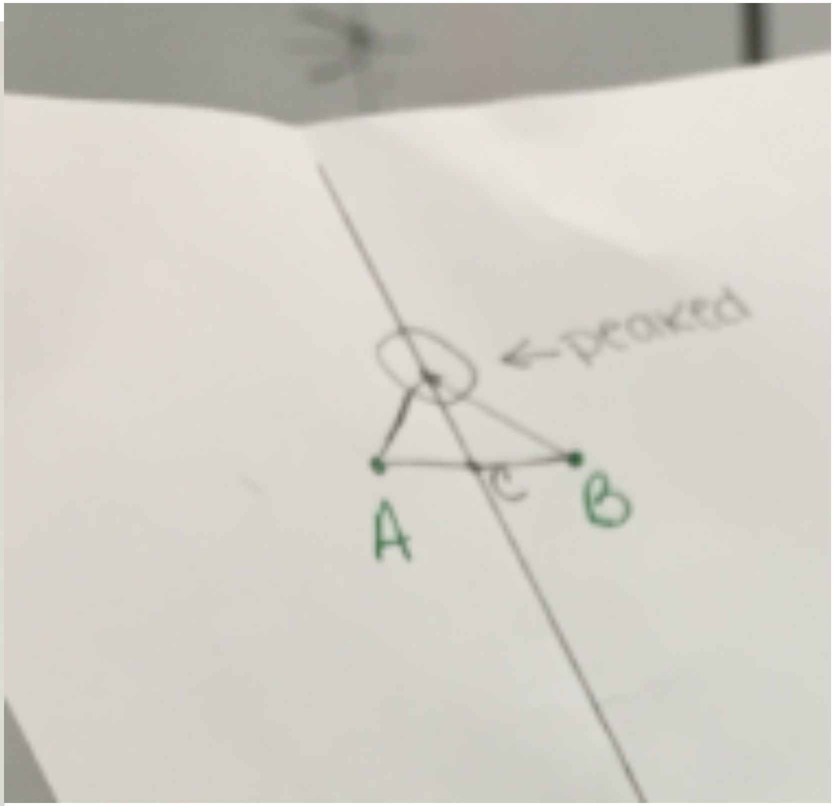
Looking at Student Work

- Look at Sharilynn’s work.
- As you watch Sharilynn’s solution unfold, make note of what she is doing.

The Building a New Playground Task V1





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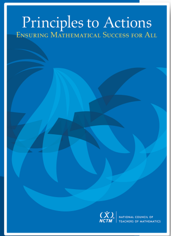

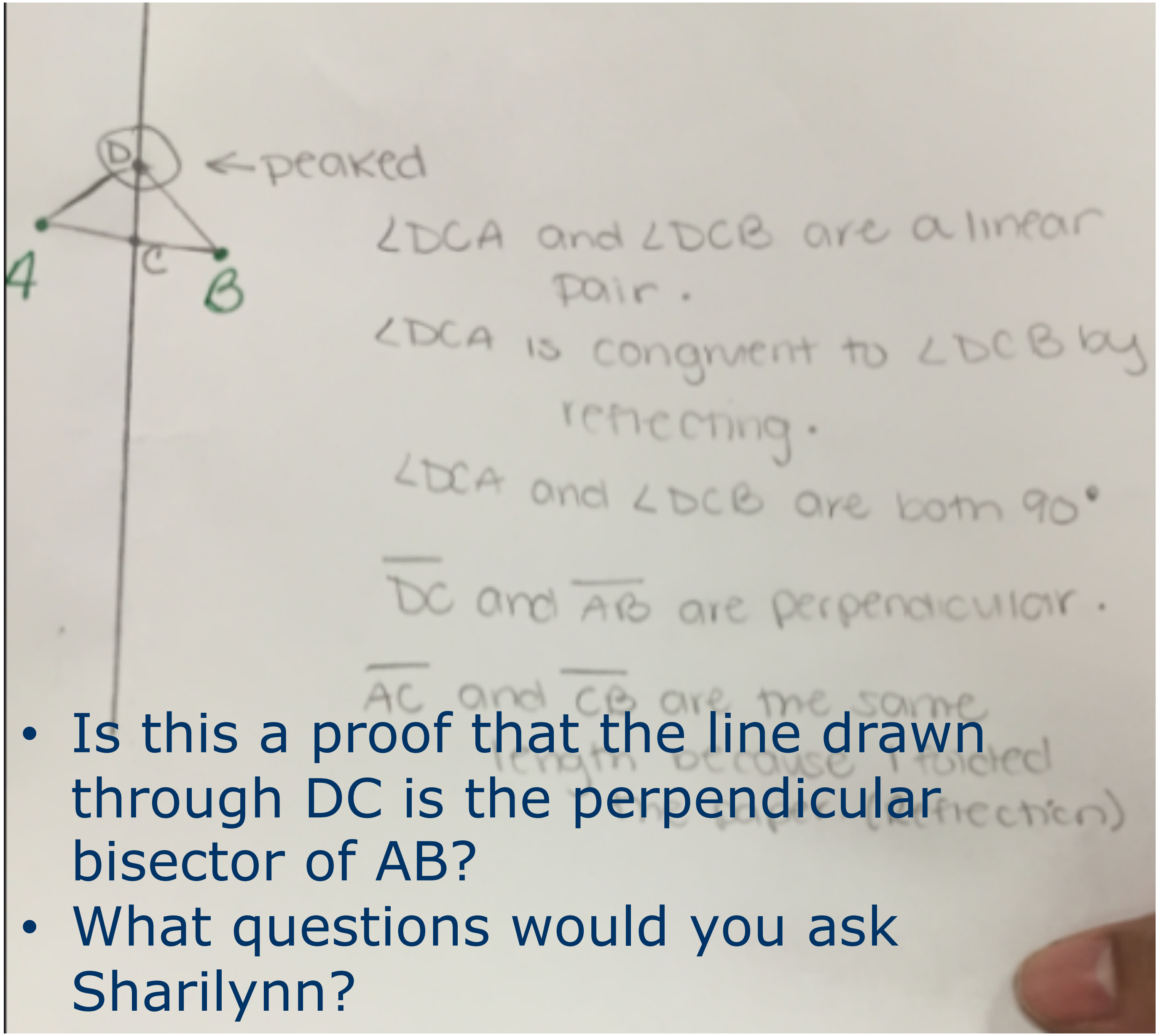

- Is this a proof that the line drawn through DC is the perpendicular bisector of AB?
- What questions would you ask Sharilynn?

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The Building a New Playground Task V2

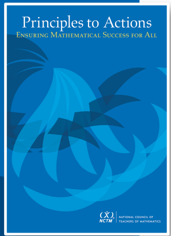
Now we are going to work on a different version of the task. As you work the task, think about differences in how you approach the task when given the information in the new task.

The Building a New Playground Task V2

The City Planning Commission is considering building a new playground. They would like the playground to be equidistant from the two elementary schools, represented by points A and B in the coordinate grid that is shown.

Part 1 – Determine at least three possible locations for the park that are equidistant from points A and B. Explain how you know that all three possible locations are equidistant from the elementary schools.

Part 2 – Make a conjecture about the location of all points that are equidistant from A and B. Prove this conjecture.



Take Time to Work the Task

- First, work on the task individually.
- Then, as a small group, consider these questions:
  - How does the addition of the grid change students' opportunities for exploring different solution paths?
  - What do you want students to learn? Which task would you use based on YOUR learning goals?

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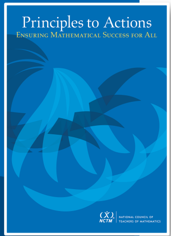
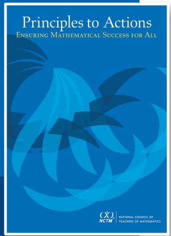


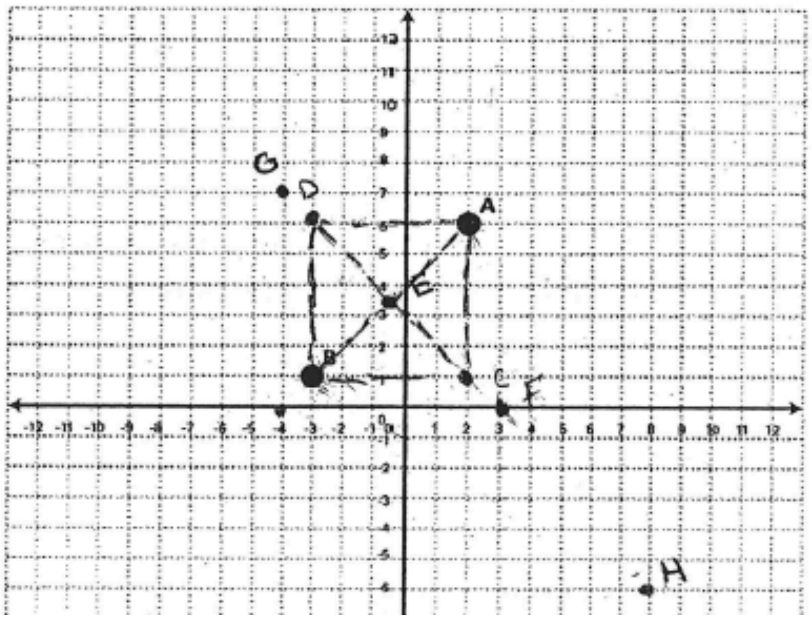
Analyzing Student Work

We are going to look at two student responses for this version of the task.



The Building a New Playground Task V2 - Faith

- you can also use the midpoint formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ to find point E.
- A square is equal distance between the points.



- Midpoint Bisector is (-0.5, 3.5)

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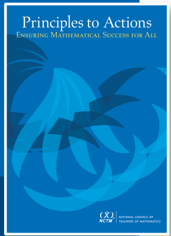
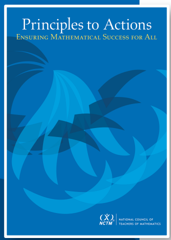


Analyzing Student Work

1. What do you think Faith knows?
2. What new methods were made available by using the grid?
3. What further steps can the Faith consider?
4. What questions can you ask now?



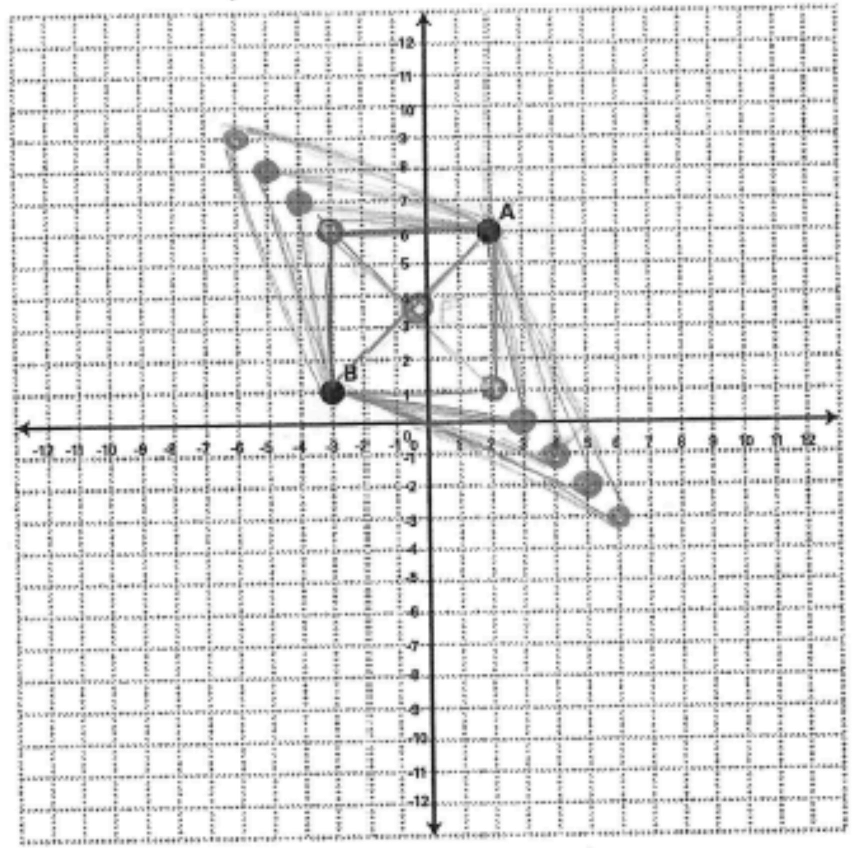
The Building a New Playground Task V2 - Leah

$\frac{X_2+X_1}{2} = \frac{-3+2}{2}$

-0.5

$\frac{Y_2+Y_1}{2} = \frac{1+6}{2}$

3.5



A(2,6)
B(-3,1)
D(3,0)

$\sqrt{(3-(-3))^2+(0-1)^2}$
$\sqrt{(6)^2+(-1)^2}$
$\sqrt{36+1}$
$\sqrt{37}$
$6.08 = \overline{BD}$

$\sqrt{(3-2)^2+(0-6)^2}$
$\sqrt{(1)^2+(-6)^2}$
$\sqrt{1+36}$
$\sqrt{37}$
$6.08 = \overline{AD}$

PART A There are an infinite number of points that are equal distance, however the closest

1 Determine at least three possible locations for the park that are equidistant from points A and B.

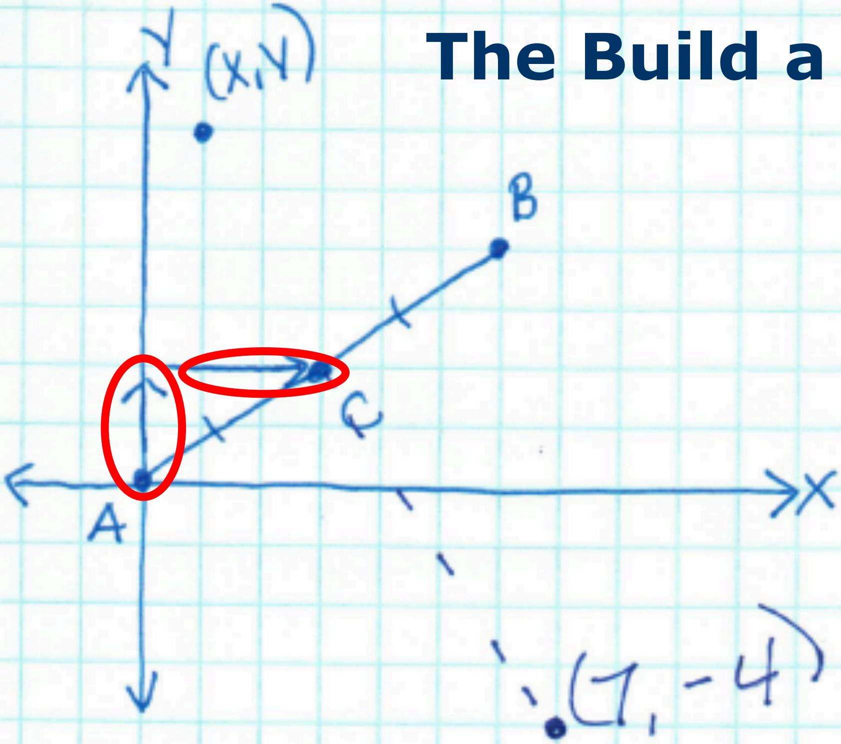
Analyzing Student Work

1. What do you think Leah knows?
2. What new methods were made available by using the grid?
3. What further steps can Leah consider?
4. Leah states there are an infinite number of points. Does she provide any proof of this?

Analyzing Student Work

This work is produced by Meggan on V1 of the task. Although the grid was not given, Meggan decided to use graph paper to work on the task.

The Build a New Playground Task V1



slope $\overline{AB} = \frac{4}{6} = \frac{2}{3}$

Midpoint up 2 over 3

$$(x-0)^2 + (y-0)^2 = (x-6)^2 + (y-4)^2$$

$$x^2 + y^2 = x^2 - 12x + 36 + y^2 - 8y + 16$$

$$12x - 36 = -8y + 16$$

$$3x - 9 = -2y + 4$$

Try a value
if $y = 2$, $x = 3$ ✓

$$3(3) - 9 = -2(2) - 4$$

$$0 = 0 \checkmark$$

Analyzing Student Work

What do you notice about Meggan's work?

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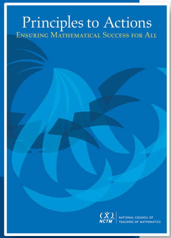


The Build a New Playground Task V1- Meggan

If $x=7$

$3(7)-9 = -2y+4$

$21-9=-2y+4$

$12=-2y+4$

$-4 \qquad -4$

$\frac{8}{-2}=\frac{-2y}{-2}$

$-4=y$

So $D(7,-4)$

Slope of $\overline{CD}$

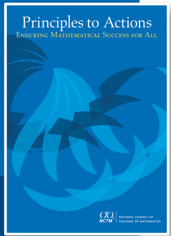
$\frac{-4-2}{7-3} = -\frac{6}{4} = -\frac{3}{2}$

Slope from A to B $= 2/3$

from C to D is $-3/2$

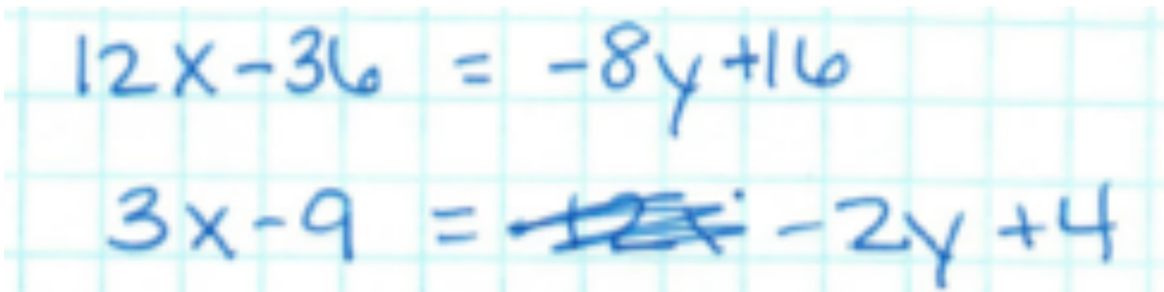
Analyzing Student Work

- What do you notice about Meggan's work?
- How might you prompt her to move on?



Promoting Productive Struggle

Meggan's teacher, initially, wanted to tell her to "solve for y" at this point.



- In what ways might "telling her" aid in or hinder productive struggle?

The Build a New Playground Task

Based on these five student responses, do you feel any of the students have "proved" their assertion?

The Playground Task V2
Video Context

School: Tyner Academy
Principal: Carol Goss
Teacher: Debra Campbell
Class: Geometry

At the time the video was filmed, Debra Campbell was a teacher at Tyner Academy in the Hamilton County School District, TN. The students are mainstream Geometry students.

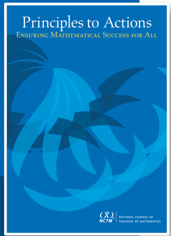
Learning Goals

Create two or three learning goals for this lesson. Be ready to share these goals.

Ms. Campbell's *Mathematics* Learning Goals

Students will understand that:

1. The points equidistant from two given points in a plane form a line;
2. The line that is formed bisects the segment connecting the two given points;
3. The line that is formed is perpendicular to the segment connecting the two given points;
4. Previously learned definitions, postulates, and theorems may be used to prove the "new" line is the perpendicular bisector of the segment connecting the two given points.



Connections to the CCSS Content Standards

Congruence	G-CO

Prove geometric theorems

C9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

Experiment with transformations in the plane

A1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: Authors.

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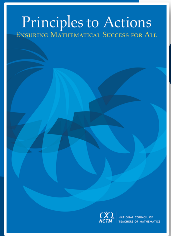

Connections to the CCSS Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.**
2. Reason abstractly and quantitatively.
3. **Construct viable arguments and critique the reasoning of others.**
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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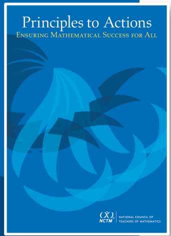
The Playground Task V2
The Context of Video Clip

Prior to the lesson:

- Students have worked with midpoint, distance, perpendicularity and related angle and linear concepts.
- Students have proved triangles congruent with SAS, SSS, ASA.

Video Clip – Ms. Campbell is working with small groups of students, asking questions to ensure students are going to continue working on the task as well as to help the students focus on their thinking and to progress toward a solution.



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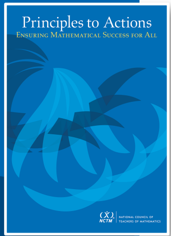

Lens for Watching the Video
First Viewing

As you watch the video, make note of what the teacher does to support student learning and engagement as they work on the task.

In particular, identify any of the *Effective Mathematics Teaching Practices* that you notice Ms. Campbell is using.

Be prepared to give examples and to cite line numbers from the transcript to support your claims.

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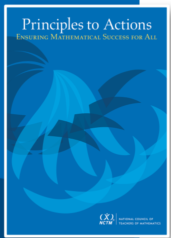

Effective
Mathematics Teaching Practices

1. Establish mathematics **goals** to focus learning.
2. Implement **tasks** that promote reasoning and problem solving.
3. Use and connect mathematical **representations.**
4. Facilitate meaningful mathematical **discourse**.
5. **Pose purposeful questions.**
6. Build **procedural fluency** from conceptual understanding.
7. **Support productive struggle in learning mathematics.**
8. **Elicit and use evidence** of student thinking.

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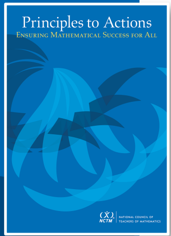

Support Productive Struggle In Learning Mathematics: Teacher and Student Actions

What are teachers doing?

- Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle.
- Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them.
- Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles.
- Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.

What are students doing?

- Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle.
- Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks.
- Persevering in solving problems and realizing that is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up.
- Helping one another without telling their classmates what the answer is or how to solve the problem.

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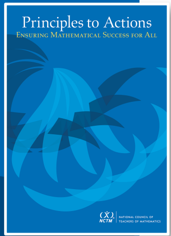

Pose Purposeful Questions

Effective Questions should:

- Reveal students' current understandings;
- Encourage students to explain, elaborate, or clarify their thinking; and
- Make the mathematics more visible and accessible for student examination and discussion.

Teachers' questions are crucial *in helping students make connections and learn important mathematics and science concepts. Teachers need to know how students typically think about particular concepts, how to determine what a particular student or group of students thinks about those ideas, and how to help students deepen their understanding.* (Weiss and Pasley, 2004)

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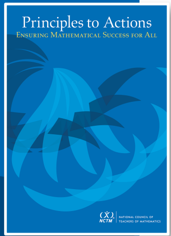
Lens for Watching the Video
Second Viewing

As you watch the video the second time, pay attention how the teacher supports students in their work and the questions that the teacher asks.

The viewing prompts on your handout provide specific questions to consider.



Lens for Watching the Video
Second Viewing

As you watch the video the second time, pay attention how the teacher supports students in their work. Specifically:

- How does the teacher intervene when incorrect or incomplete strategies are used?
- How are students dealing with frustrations when working on the task?
- Are the students making connections among the mathematical representations?

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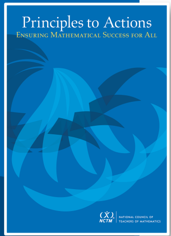
Lens for Watching the Video Second Viewing

As you watch the video this time, pay attention to the questions the teacher asks. Specifically:

- Which questions, if any, are gathering information?
- Which questions, if any, are probing thinking? What do the questions reveal about students' current understandings
- Which questions make mathematics more visible and accessible for student examination and discussion?
- Which questions, if any, encourage reflection and justification? How do students answer those questions?



How might your apply what you have learned about the effective mathematics teaching practices to your own classroom instruction?

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