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# Moving to Action: Mathematics Teaching Practices to Support Diverse Learners

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Middle Grades Master Class

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# Today's Action-Packed Agenda

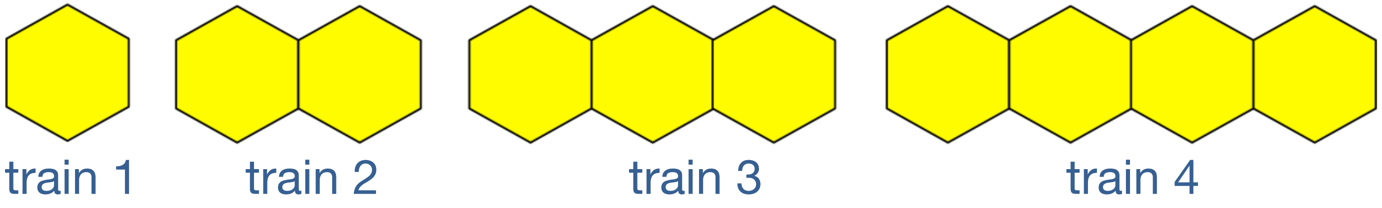
- Solve and discuss the Hexagon Task
- NCTM's Effective Mathematics Teaching Practices: A framework for supporting instructional change
- The Case of Barbara Peterson (narrative)
- The Case of Patricia Rossman (video)
- Next steps for YOUR TEAM

# Solving and Discussing the Hexagon Task

A task of high cognitive demand

# The Hexagon Task

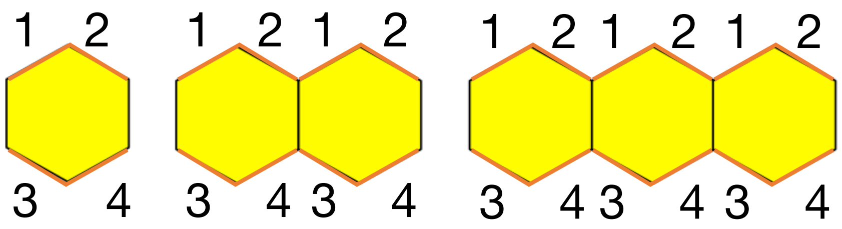
Trains 1, 2, 3, and 4 are the first 4 trains in the hexagon pattern. The first train in this pattern consists of one regular hexagon. For each subsequent train, one additional hexagon is added.



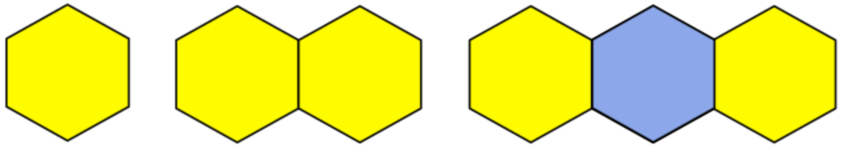

1. Compute the perimeter for each of the first four trains;
2. Draw the fifth train and compute the perimeter of the train;
3. Determine the perimeter of the 25th train without constructing it;
4. Write a description that could be used to compute the perimeter of any train in the pattern; and
5. Determine which train has a perimeter of 110.

# Hexagon Task: Some possible solutions

A Tops & Bottoms, plus 2 ends
$y = 4x + 2$

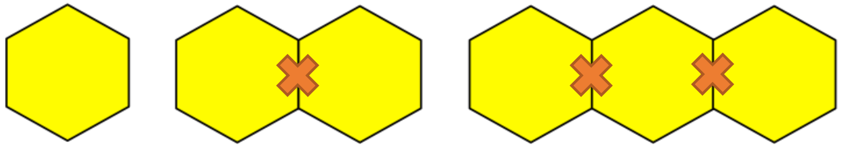



B Insides and Outsides
$y = 4(x - 2) + 10$



The two outside hexagons each contribute 5 sides
The inside hexagons each contribute 4
Number of inside hexagons is train number minus 2

C Shared sides subtracted
$y = 6\underline{x} - 2(x - 1)$



Each hexagon contributes 6 sides
For each new hexagon past train 1, there is a pair of inside sides that have to be subtracted
The number of shared pairs is the train number minus 1

# The Effective Mathematics Teaching Practices

A framework for supporting all students, with particular attention to our struggling learners

# Principles to Mathematical Actions: Ensuring Success for All

The primary purpose of Principles to Actions is to fill the gap between the adoption of rigorous standards and the enactment of practices, policies, programs, and actions required for successful implementation of those standards.


NCTM (2014). Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: NCTM.

# Organization of P2A

**Essential Elements of Effective Math Programs**

1. Teaching and Learning
2. Access and Equity
3. Curriculum
4. Tools and Technology
5. Assessment
6. Professionalism

**For each element…**

- Productive and Unproductive Beliefs are Listed
- Obstacles to Implementing the Principle are Outlined
- Overcoming the Obstacles
- Taking Action
    - Leaders and Policymakers
    - Principals, Coaches, Specialists, Other School Leaders
    - Teachers

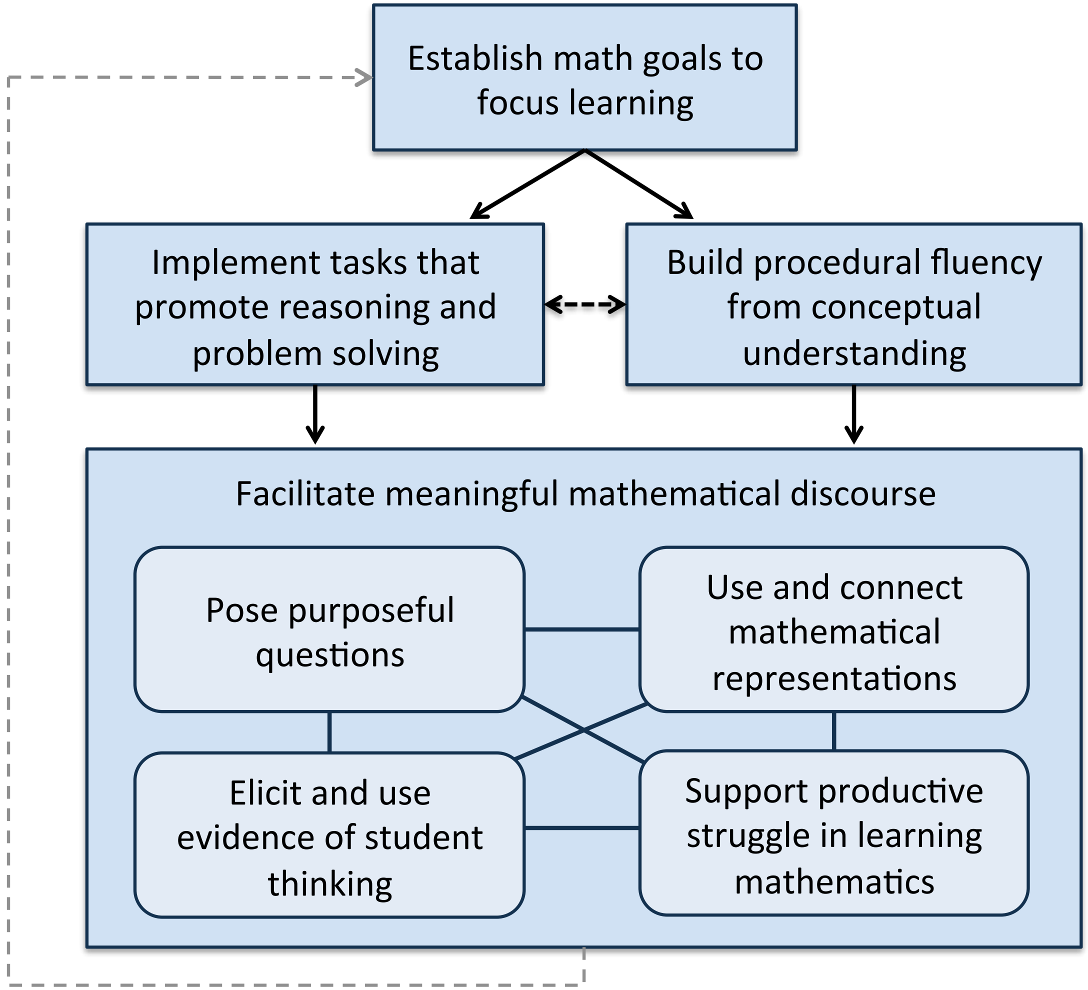
# Teaching and Learning Principle

An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.

# Effective Mathematics Teaching Practices

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical **discourse.**
5. Pose purposeful **questions.**
6. Build procedural fluency from conceptual understanding.
7. Support **productive struggle** in learning mathematics.
8. Elicit and use evidence of student thinking.

# Effective Mathematics Teaching Practices



# The Case of Barbara Peterson

# The Case of Barbara Peterson

Read the narrative Case of Barbara Peterson, which describes a teacher implementing the Hexagon Pattern Task.

As you read, identify any of the Effective Mathematics Teaching Practices that you notice Ms. Peterson using.

Be prepared to give examples and to cite line numbers to support your claims.

# Effective Mathematics Teaching Practices in The Case of Barbara Peterson

1. Establish mathematics **goals** to focus learning.
2. Implement **tasks** that promote reasoning and problem solving.
3. Use and connect mathematical **representations.**
4. Facilitate meaningful mathematical **discourse.**
5. Pose purposeful **questions.**
6. Build **procedural fluency** from conceptual understanding.
7. Support **productive struggle** in learning mathematics.
8. **Elicit and use evidence** of student thinking.

# Establish Mathematics Goals To Focus Learning

Learning Goals should:

- Clearly state what it is students are to learn and understand about mathematics as the result of instruction;
- Be situated within learning progressions; and
- Frame the decisions that teachers make during a lesson.

Formulating clear, explicit learning goals sets the stage for everything else.

(Hiebert, Morris, Berk, & Janssen, 2007, p. 57)

# Goals to Focus Learning

Ms. Peterson's goal for students' learning:

1. variables can be used to represent two quantities that change in the relationship to each other;
2. there are different but equivalent ways of writing an explicit rule; and
3. connections can be made between different representational forms – tables, graphs, equations, words, and pictures.

How do her goals align with this teaching practice?

# Goals to Focus Learning

- Is your goal visible throughout your lesson in both teacher and student actions?
- How do students know they are making progress towards the goal?
- What will students say when they get home about what they did in math today?
    - We worked in groups
    - We used manipulatives
    - We generalized a linear pattern

# Implement Tasks that Promote Reasoning and Problem Solving

Mathematical tasks should:

- Provide opportunities for students to engage in exploration or encourage students to use procedures in ways that are connected to concepts and understanding;
- Build on students' current understanding; and
- Have multiple entry points.

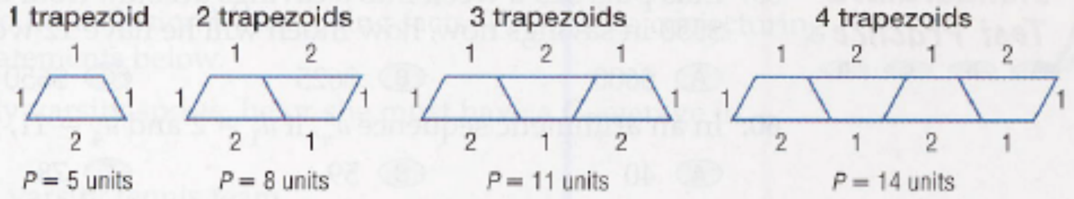
There is no decision that teachers make that has a greater impact on students' opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages students in studying mathematics.

(Lappan & Briars, 1995)

# Implement Tasks that Promote Reasoning and Problem Solving

- How is the Hexagon task similar to or different from the Trapezoid task?
- Which one is more likely to promote problem solving?

**GEOMETRY** For Exercises 45 and 46, use the diagram below that shows the perimeter of the pattern consisting of trapezoids.




**45.** Write a formula that can be used to find the perimeter of a pattern containing $n$ trapezoids.

**46.** What is the perimeter of the pattern containing 12 trapezoids?

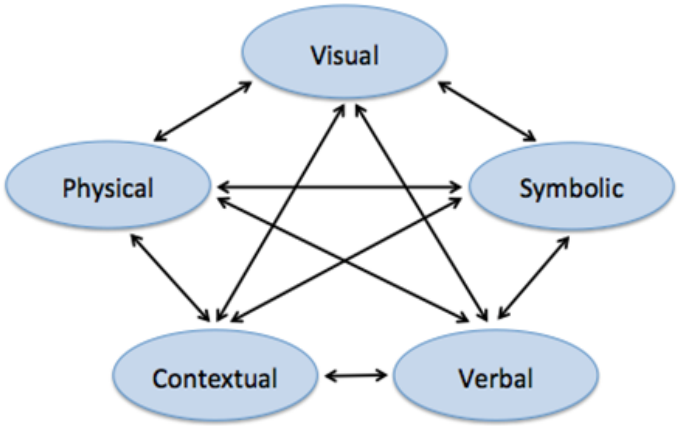
# Implement Tasks that Promote Reasoning and Problem Solving

- There are a wealth of sources of good mathematical tasks in the world today!
  - Student-centered curriculum materials
  - Curated websites like illustrativemathematics.org, threeacts.mrmeyer.com, robertkaplinsky.com
- Adapting tasks in your own resources to increase the cognitive demand is not as difficult as you think!
  - See Geoff Krall's Adaptation presentation for examples
  - Also see the work of Dan Meyer & Peg Smith

# Use and Connect Mathematical Representations

Different Representations should:

- Be introduced, discussed, and connected;
- Focus students' attention on the structure or essential features of mathematical ideas; and
- Support students' ability to justify and explain their reasoning.




Strengthening the ability to move between and among these representations improves the growth of children's concepts.

(Lesh, Post & Behr, 1987)

# Use and Connect Mathematical Representations

- What different representations were used and connected in Ms. Peterson's class?
- How might her students benefit from making these connections?

# Facilitate Meaningful Mathematical Discourse

Mathematical Discourse should:

- Build on and honor students' thinking;
- Provide students with the opportunity to share ideas, clarify understandings, and develop convincing arguments; and
- Advance the mathematical learning of the whole class.

Discussions that focus on cognitively challenging mathematical tasks…are a primary mechanism for promoting conceptual understanding of mathematics (Hatano and Inagaki 1991; Michaels, O'Connor and Resnick 2008).

(Smith, Hughes, Engle and Stein 2009, p. 549)

# Facilitate Meaningful Mathematical Discourse

- What did Ms. Peterson do (before or during the discussion) that may have positioned her to engage her students in a productive discussion?
- What moves do Ms. Peterson and her students make that promote meaningful mathematical discourse?

# Facilitate Meaningful Mathematical Discourse

### 5 Practices for Orchestrating Productive Discussions

- **Anticipating**
- Monitoring
- Selecting
- Sequencing
- Connecting

Smith & Stein (2011)


### Teacher Discourse Moves

- Waiting
- Inviting student participation
- Revoicing
- Asking students to revoice
- Probing a student's thinking
- Creating opportunities to engage with another's reasoning

Herbel-Eisenmann, Steele, & Cirillo (2013)

# Pose Purposeful Questions

Effective Questions should:

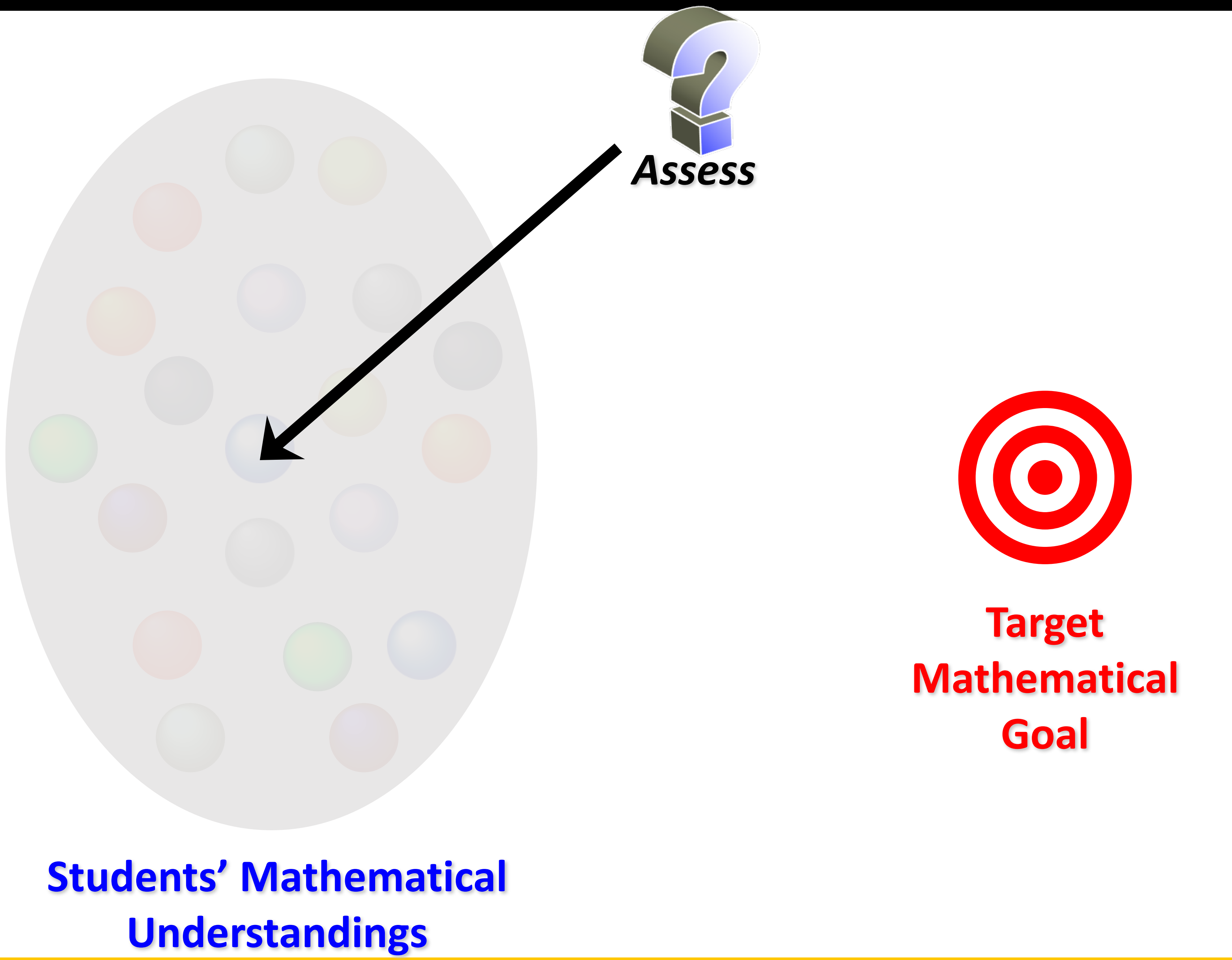
- Reveal students' current understandings;
- Encourage students to explain, elaborate, or clarify their thinking; and
- Make the mathematics more visible and accessible for student examination and discussion.

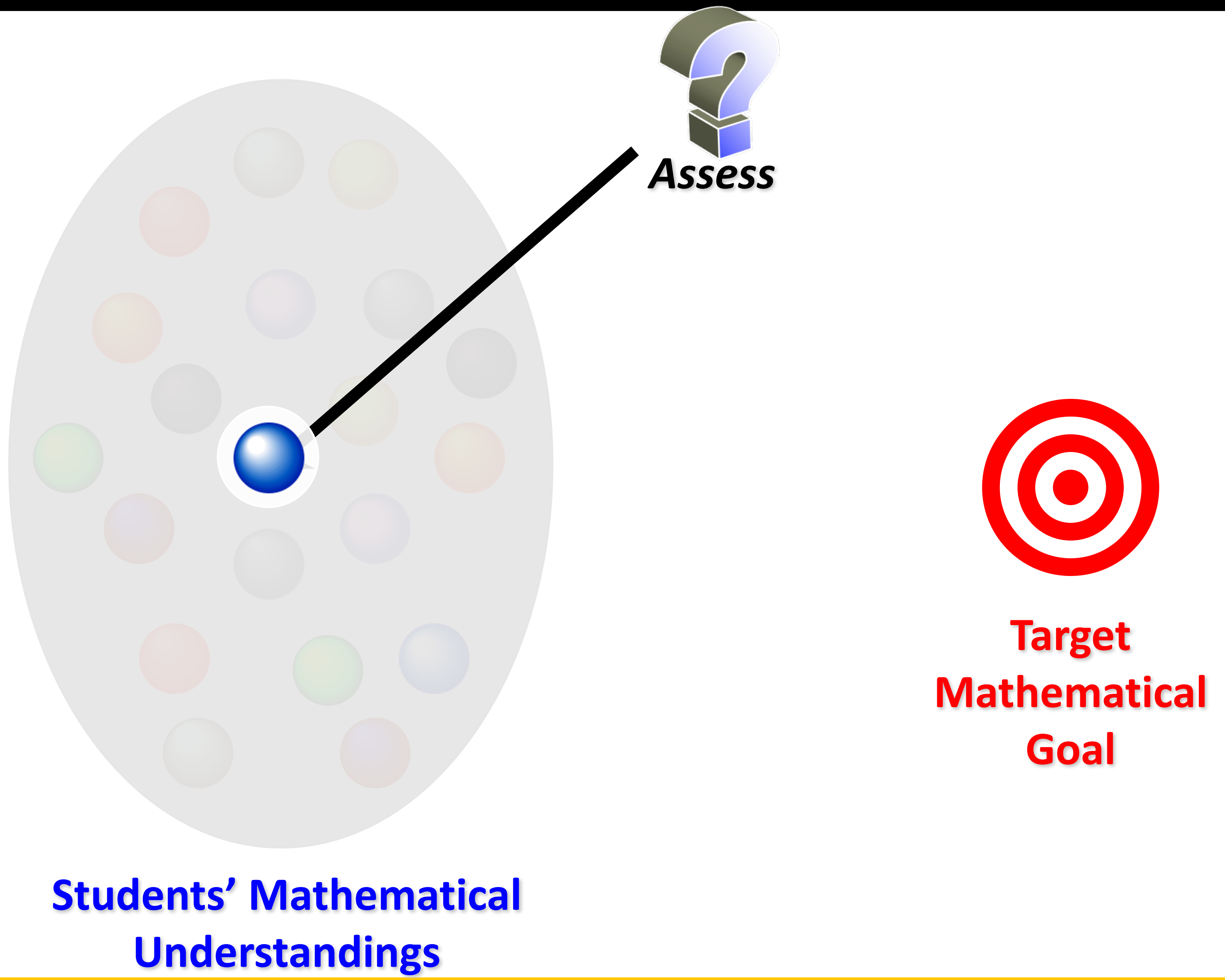
Teachers' questions are crucial in helping students make connections and learn important mathematics and science concepts. Teachers need to know how students typically think about particular concepts, how to determine what a particular student or group of students thinks about those ideas, and how to help students deepen their understanding.

(Weiss & Pasley, 2004)

# Pose Purposeful Questions

- What do you notice about the questions that Ms. Peterson asked on lines 36-42 and 55-75?
- What purpose did her questions serve?


Assess
Target Mathematical Goal
Students' Mathematical Understandings


Assess
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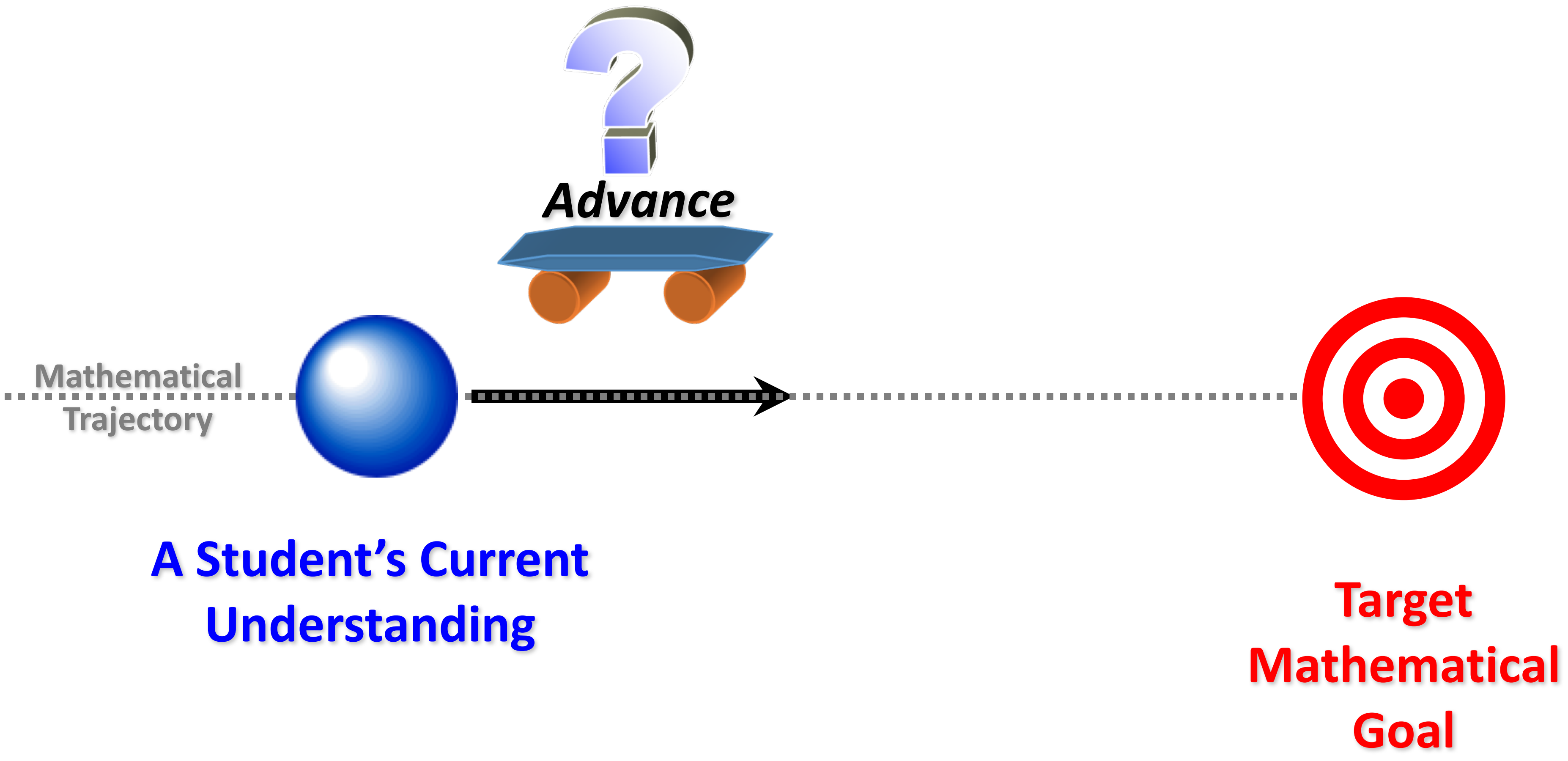
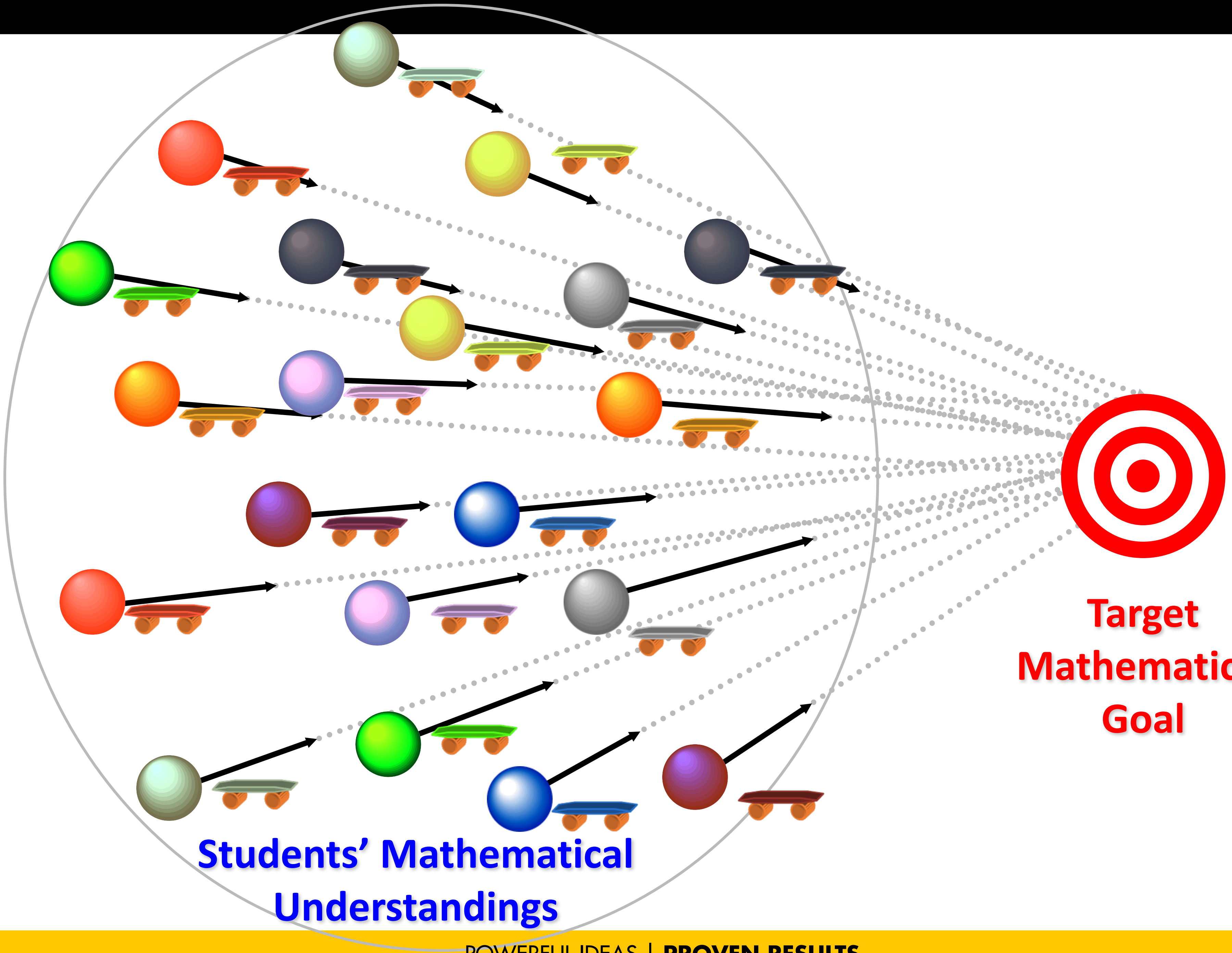

Advance
Mathematical
Trajectory
A Student's Current
Understanding
Target
Mathematical
Goal


Target
Mathematical
Goal
Students' Mathematical
Understandings

# Build Procedural Fluency from Conceptual Understanding

Procedural Fluency should:

- Build on a foundation of conceptual understanding;
- Result in generalized methods for solving problems; and
- Enable students to flexibly choose among methods to solve contextual and mathematical problems.

Students must be able to do much more than carry out mathematical procedures. They must know which procedure is appropriate and most productive in a given situation, what a procedure accomplishes, and what kind of results to expect. Mechanical execution of procedures without understanding their mathematical basis often leads to bizarre results.

(Martin 2009, p. 165)

# Build Procedural Fluency from Conceptual Understanding

What might we expect the students in Ms. Peterson's class to be able to do after they have had the opportunity to develop an understanding of how linear functions model situations, what the slope means in a context, and how slope is represented in a table, graph and equation?

- Be able to create equations that model situations by connecting key aspects of a situation with parameters such as slope and y-intercept.
  (As opposed to creating a decontextualized table of values and finding the difference in successive y values when x is incremented by 1, then plugging it in as the *m* in $y = mx + b$.)
- Identify what slope is given any representational form and explain how the different representations are connected.
  (As opposed to only being able to find the slope using two points and the formula and not being able to relate it to anything.)

# Support Productive Struggle in Learning Mathematics

Productive Struggle should:

- Be considered essential to learning mathematics with understanding;
- Develop students' capacity to persevere in the face of challenge; and
- Help students realize that they are capable of doing well in mathematics with effort.

By struggling with important mathematics we mean the opposite of simply being presented information to be memorized or being asked only to practice what has been demonstrated.

(Hiebert and Grouws 2007, pp. 387-88)

# Support Productive Struggle in Learning Mathematics

- How did Ms. Peterson support students when they struggled?

# Elicit and Use Evidence of Student Thinking

Evidence should:

- Provide a window into students' thinking;
- Help the teacher determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(Wiliam 2007, pp. 1054; 1091)

# Elicit and Use Evidence of Student Thinking

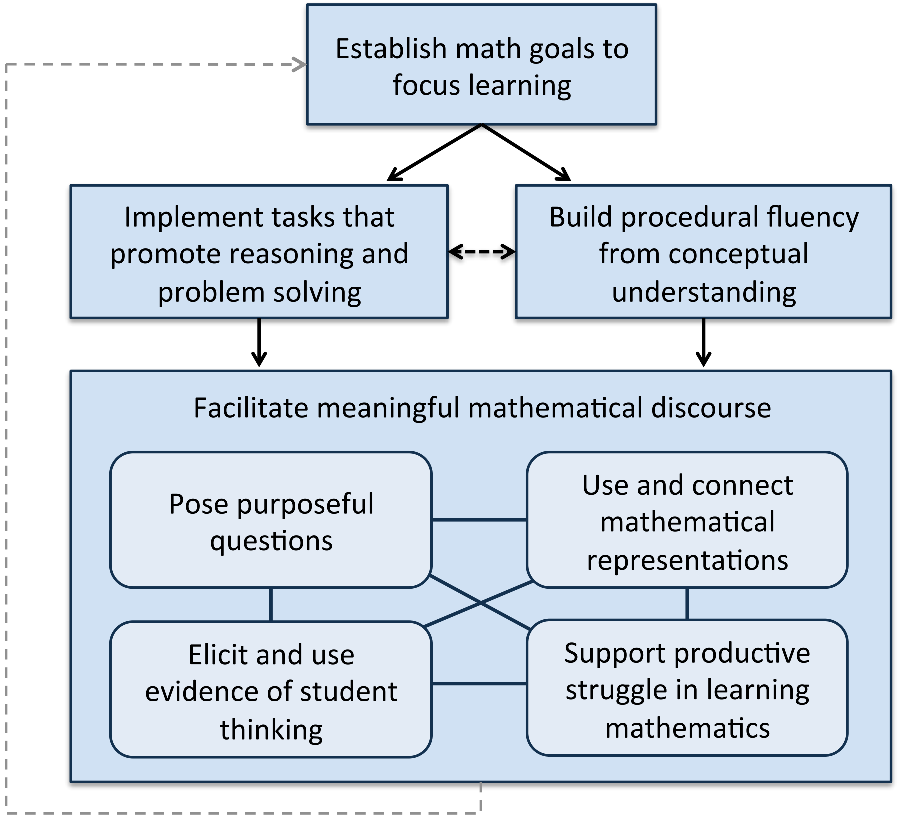
- To what extent did Ms. Peterson elicit students' thinking?
- To what extent did (or could) Ms. Peterson use the evidence to inform her instruction?

# Effective Teaching Means Student *Activity*, Not Just Engagement

"If your students are going home at the end of the day less tired than you are, the division of labor in your classroom requires some attention."

(Wiliam, 2011)

# Effective Mathematics Teaching Practices: Supporting Struggling Students




**Effective Mathematics Teaching Practices**
**"Building a Teaching Framework"**

For each of the eight practices:

- What **challenges** do you see with this practice in supporting struggling students?
- What **opportunities** do you see for this practice in supporting struggling students?

# A Closer Look at Productive Struggle: The Case of Mrs. Rossman

- Mrs. Rossman's sixth grade students (who are in a dual language program) are working on the hexagon task. She wants students to represent two quantities that change in relationship to one another.
- One group of students was struggling to generalize, just like Group 2 in Ms. Peterson's class did (lines 36-39).
- As you watch the video, note what Mrs. Rossman does to support her students productive struggle, and what other effective mathematics teaching practices were at play.

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# Five Teacher Responses to Productive Struggle

Better chance of productive struggle

- Unfocused or vague
  *provides a suggestion that is too general to be supportive*
- Telling
  *supplying students with information that removes the struggle*
- Directed guidance
  *redirecting students to another strategy consistent with the teacher's thinking*
- Probing guidance
  *determining what the student is thinking, encouraging self-reflection, offering ideas based on student thinking*
- Affordance
  *asking the student to articulate what they have done, encouraging continued effort with limited intervention, allowing students time to work*

Adapted from Warshauer, 2015 and Smith, Steele, & Raith, 2017

# Access and Equity

Although some research supports grouping gifted and talented students in homogeneous groups to maximize their learning (Delcourt et al. 1994), research also shows that the learning of students assigned to lower-ability groups is depressed, regardless of their ability levels (Stiff, Johnson, and Akos 2011). In addition, once students are placed in low-level or “slow” math groups, they are very likely to remain in those groups until they leave school (Boaler 2008; Ellis 2008). When middle level students thought to be “at risk" in mathematics are placed in grade-level mathematics courses and provided the support necessary to be successful in those courses, their achievement gains are greater, and they are more likely to enroll in upper level courses in the following years, than when they are placed in lower-ability math courses (Boaler and Staples, 2008; Burris, Heubert, and Levin 2006). Further, evidence suggests that high-achieving students in heterogeneous classes are not statistically different from homogeneously tracked students in achievement and participation in Advanced Placement (AP) mathematics courses (Burris, Heubert, and Levin 2006; Staples 2008).

(NCTM, 2014, p. 62)

# Next Steps

- How can you integrate opportunities for using the eight effective mathematics teaching practices in your classroom **this year**?
- How will you instigate conversations with your students and colleagues about the importance of productive struggle in supporting struggling students?

# Thank you!

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