2 = 1 + 1

and other

Compositions

with Fawn Nguyen
and Joshua Zucker
@fawnpnguyen, @joshuazucker

What is a Composition?

- 2 = 1 + 1 are the two legal ways to make 2.
- There are four legal ways to make 3. What are they?
- 3 = 2 + 1 = 1 + 2 = 1 + 1 + 1.
- So what are the rules?
- Only positive integers.
- Only addition.
- Or: “The total of a list of positive integers”

The Challenge

- How many ways to make 10?
- Is there a number you're sure is too small?
  A number you're sure is too big?
  What's your best guess at the answer?
- How do you respond to a hard problem?
  - Do an easier one!
  - Find a way to strategize, organize.
  - Patience!

Easier problem: 4

- How many legal ways to make 4?
- How do you know your list is complete and doesn't have any duplicates?
- Organization!
- $4 = 3+1 = 2+2 = 1+3$
  $= 2+1+1 = 1+2+1 = 1+1+2$
  $= 1+1+1+1$
- That's one way to organize, brainstorm more now!

Ways to Organize

- How many parts (as we did with 4).
- First part (or last part).
- Size of largest part.
- Size of smallest part.
- How many 1s are used.
- How many different parts
- More ideas?

Organizing by first part

- 1. Well, there's one way, first part is 1.
- 2. One with 1 first, 1+1.
  One with 2 first, 2.
- 3. Two with 1 first, 1+1+1 and 1+2.
  One with 2 first, 2+1.
  One with 3 first, 3.
- 4. Four, two, one, one.
- A pattern? Does it continue? Why does it happen that way?

First part

	1	2	3	4	5	6	7
1	1						
2	1	1					
3	2	1	1				
4	4	2	1	1			
5							
6							
7							

Easier problem: 5

- How many start with 1? No, too hard.
- How many start with 5? OK, good.
- How many start with 4? Why?
- How many start with 3?
  5 = 3+
  = 3+
- 5 = 3+2
  = 3+1+1

Easier problem: 5

- Start with 5 = 2+ … what do we need to finish?
- Right, 3 more. And how many ways are there to do it?
- We can recycle our previous results! So doing the other easier problems actually directly helps us do the harder ones.
- There are four ways to make 3, so there are four ways to make 5 starting with 2.
- And 1?

First part

	1	2	3	4	5	6	7
1	1						
2	1	1					
3	2	1	1				
4	4	2	1	1			
5	8	4	2	1	1		
6							
7							

First part

	1	2	3	4	5	6	7
1	1						
2	1	1					
3	2	1	1				
4	4	2	1	1			
5	8	4	2	1	1		
6	16	8	4	2	1	1	
7	32	16	8	4	2	1	1

Problem solved!

- So, how many ways to make 10?
- Indeed, 512.
- And look at all the strategies we've picked up along the way already: Easier problem. Organize. Patience. Recycle.
- So, the real lesson here: don't stop when you have an answer. Explore! Create questions! Solve it a different way!

How Many Parts

	1	2	3	4	5	6	7
1	1						
2	1	1					
3							
4							
5							
6							
7							

By how many parts

- 1. Well, there's one way, one part: 1.
- 2. One with 2 parts, 1+1.
  One with 1 part, 2.
- 3. One with 3 parts, 1+1+1.
  Two with 2 parts, 2+1, 1+2.
  One with 1 part, 3.
- 4 = 3+1 = 2+2 = 1+3
  = 2+1+1 = 1+2+1 = 1+1+2
  = 1+1+1+1.

How Many Parts

	1	2	3	4	5	6	7
1	1						
2	1	1					
3	1	2	1				
4	1	3	3	1			
5							
6							
7							

By how many parts

- Recognize the pattern?
- Wonder if 5 with 3 parts will be 6.
- How can we recycle now?
- 4 with 2 parts: end with a +1.
- 4 with 3 parts: how to turn it into 5 with 3 parts?

Recycle!

We still need to fit
…
Wait, what are the other ways? Are there really three more?

3+1	3+1**+1**
2+2	2+2**+1**
1+3	1+3**+1**
2+1+1	
1+2+1	
1+1+2	

Recycle!

We still need to fit
1+1+3
1+2+2
2+1+2
but which one goes with which, and why?

3+1	3+1+1
2+2	2+2+1
1+3	1+3+1
2+1+1	
1+2+1	
1+1+2	

Recycle!

3+1	3+1+1
2+2	2+2+1
1+3	1+3+1
2+1+**1**	2+1+**2**
1+2+**1**	1+2+**2**
1+1+**2**	1+1+**3**

Recycle, caveman style!

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Recycle, caveman style!

Now we can see that the caveman style of mathematics has its advantages even today.

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Largest Part

	1	2	3	4	5	6	7
1	1						
2	1	1					
3	1	2	1				
4							
5							
6							
7							

	1	2	3	4	5	6	7
1	1						
2	1	1					
3	1	2	1				
4	1	4	2	1			
5	1		5	2	1		
6	1			5	2	1	
7	1				5	2	1

Conclusion

- We can count how many ways to make any number as a list of positive integers.
- Along the way we encounter powers of 2, Pascal's triangle, and much more!
- Strategies: Easier problem, organization, and above all, recycle.
- Creating new problems can be the best way to deepen your understanding.