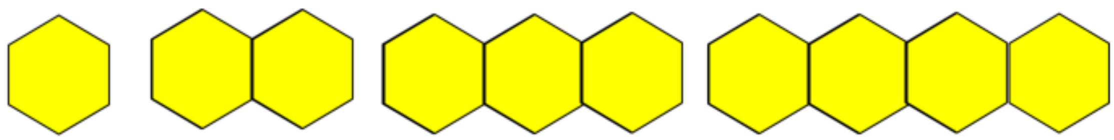
Hexagon Task[1]

Trains 1, 2, 3 and 4 (shown below) are the first 4 trains in the hexagon pattern. The first train in this pattern consists of one regular hexagon. For each subsequent train, one additional hexagon is added.



Train 1 Train 2 Train 3 Train 4

1. Compute the perimeter for each of the first four trains.
2. Draw the fifth train and compute the perimeter of the train.
3. Determine the perimeter of the 10th train without constructing it.
4. Write a description that could be used to compute the perimeter of any train in the pattern.
5. Determine which train has a perimeter of 110.

[1] This task was adapted from *Visual Mathematics Course 1, Lessons 16-30* published by the Math Learning Center. Copyright © 1995 by The Math Learning Center, Salem, Oregon.

Effective Mathematics Teaching Practices
Establish mathematics goals to focus learning. *Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.*
Implement tasks that promote reasoning and problem solving. *Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.*
Use and connect mathematical representations. *Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.*
Facilitate meaningful mathematical discourse. *Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.*
Pose purposeful questions. *Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.*
Build procedural fluency from conceptual understanding. *Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.*
Support productive struggle in learning mathematics. *Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.*
Elicit and use evidence of student thinking. *Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.*

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

Writing Team: Steve Leinwand, Daniel J. Brahier, DeAnn Huinker, Robert Q. Berry III, Frederick L. Dillon, Matthew R. Larson, Miriam A. Leiva, W. Gary Martin, and Margaret S. Smith.
http://www.nctm.org/principlestoactions

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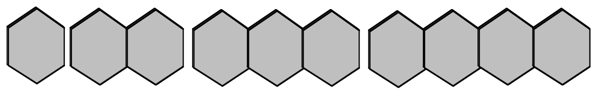
Exploring Linear Functions: *The Case of Barbara Peterson*[2]

Ms. Peterson wanted her eighth grade students to understand three key ideas about linear functions: 1) variables can be used to represent two quantities that change in the relationship to each other; 2) that there are different but equivalent ways of writing an explicit rule; and 3) that connections can be made between different representational forms – tables, graphs, equations, words, and pictures. She selected the Hexagon Pattern Train Task because it aligned with her goals for the lesson and with state standards, it had multiply entry points and solution paths, and it would challenge her students to think and reason.

Trains 1, 2, 3, and 4 are the first 4 trains in the hexagon pattern. The first train in this pattern consists of one regular hexagon. For each subsequent train, one additional hexagon is added.



train 1 train 2 train 3 train 4

1. Compute the perimeter for each of the first four trains;
2. Draw the fifth train and compute the perimeter of the train;
3. Determine the perimeter of the 10th train without constructing it;
4. Write a description that could be used to compute the perimeter of any train in the pattern; and
5. Determine which train has a perimeter of 110.

Ms. Peterson told students that questions 1-3 were to help them get started if they needed it, question 5 was for students who finished quickly, but that question 4 would be the focus of the whole class discussion. She reminded students that they could use any of materials (hexagon shapes, copies of the hexagon trains, colored pencils, calculators) that were available on their tables.

As students began working in their groups, Ms. Peterson walked around the room stopping at different groups to listen in on their conversations and to ask questions as needed. When groups struggled to get started, she asked: "What would train 5 look like? How much bigger is the perimeter of train five than the perimeter of train 4? What is different as you move from one train to the next? How does the pattern seem to be growing?" She also encouraged students to use the hexagon shapes to build new trains and to make observations about what changes and what remains the same as the trains get bigger. When groups determined a correct algebraic representation she pressed them to explain what each part of the equation meant, to relate each part of the equation to the picture, and to explain how they knew it would always work.

Ms. Peterson spent more time with Groups 2 and 5. Group 2 had identified the growth rate as +4 but was not able to find the perimeter of a train without knowing the perimeter of the train that preceded it. She asked: "Why are you adding 4? Where is the 4 in the picture? Can you predict the number of 4's that you are adding each time?'' Group 5 had reasoned that since a hexagon has 6 sides that every time you add one more hexagon you add 6 more sides. She asked: "When you use your rule to find the perimeter in train 4, do you get the same answer as you did when you counted? Why not? What gets counted when you multiply by 6?"

As she visited the groups, Ms. Peterson made note of the strategies students were using (see reverse side) so she could decide which groups she wanted to have present their work. Each presenting group would be expected to explain what they did and why, how their equation connected to the visual arrangement of hexagons, and to answer questions posed by their peers. Group 3 would go first since their table would be accessible to all groups. Group 1 would go next because the equation they created modeled the verbal description given by Group 3 but was derived from the picture rather than from numbers in a table. Group 5 would then present a different equation, also derived from the picture, and discuss how they came to see that counting all of the sides did not work.

[2]This case, written by Margaret Smith (University of Pittsburgh), was inspired by lessons planned and taught by Timothy Booth, a graduate student at the University of Pittsburgh and other teachers with whom she has worked over the last decade. It is intended to support the Teaching and Learning Guiding Principle in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014).

The presentations would conclude with Group 2. Below is an excerpt from the discussion that took place around the explanation presented by Group 2.

Michael: If you look at Train 4 we saw that there were five sides on each end of the train that counted in the perimeter and that the two hexagons in the middle each had 4 sides that counted. So we had 5 + 4 + 4 + 5 which is 18.

Ms. P.: So suppose we were talking about the 10th train. Can someone else explain how we could use Group 2's method to find the perimeter of the 10^{th} train? *(Several students raise their hands and the teacher calls on Kelsey.)*

Kelsey: For Train 10 it would 5 + 5 for the two ends and then there would be 8 4's in the middle. So it would be 32 + 10 which is 42.

Brian: How did you know how many 4's would be needed?

Kelsey: I pictured it in my head and if I took off the two ends then there would be 8 hexagons left.

Ms. P.: I am wondering if we could generalize the number of hexagons that are between the two ends of any train. Take two minutes and turn and talk to the person sitting next to you. *(After two minutes the teacher continues.)* Amber? What did you and Sara decide?

Amber: Well we think that the number of hexagons in the middle is h – 2. Then you would need to multiply by 4 to get the total number of sides that the ones in the middle contribute to the perimeter. That is 4(n-2).

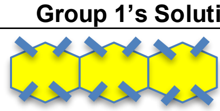
Ms. P.: What do others think? *(Students are nodding their heads or giving a "thumbs up".)*

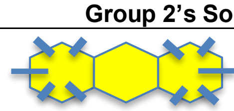
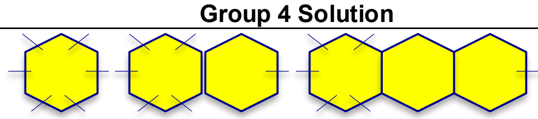
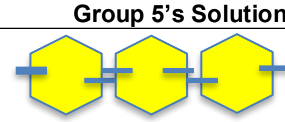
Ms. P.: If we use 4(n-2) like Amber and Jackie are suggesting, will we get the correct perimeter?

Sara: No. You need to add on the 5 for the first hexagon and the 5 for the last hexagon. So it has to be P= 4(n-2) + 10.

Ss: YES!

With only five minutes left in the class, Ms. Peterson asked students to picture in their minds what the graph of this function they had been working on would look like, to sketch the graph, and explain why they think their sketch made sense. She would use this information to see what students understood about relationship between representations and to launch class the following day. She also planned to talk about how there could be three (or more) equations for the same function.

<table>
<tr><th>Group 1's Solution</th><th>Group 2's Solution</th><th>Group 3's Solution</th></tr>
<tr>
<td>
Each hex has 2 sides on the top and 2 on the bottom that count. So that's 4. Then you need to add the two on the ends.

It is P = 4h +2</td>
<td>
There are five sides on each end of the train. Then every time you add another hexagon there are 4 more sides on the inside – 2 on the top and 2 on the bottom.</td>
<td><table><tr><th>Train #</th><th>Perimeter</th></tr><tr><td>1</td><td>6</td></tr><tr><td>2</td><td>10</td></tr><tr><td>3</td><td>14</td></tr><tr><td>4</td><td>18</td></tr><tr><td>5</td><td>22</td></tr><tr><td>6</td><td>26</td></tr><tr><td>7</td><td>30</td></tr><tr><td>8</td><td>34</td></tr></table>The perimeter of each train is 4 more. So if you multiply the number of hexagons by 4 you need to add on 2 to get the right perimeter.</td>
</tr>
<tr><th colspan="2">Group 4 Solution</th><th>Group 5's Solution</th></tr>
<tr>
<td colspan="2">
The first hexagon train has six sides. When you go to bigger trains the right side of the first hexagon moves to the end of the train. Each of the other hexagons in a train adds 4 sides to the perimeter.
P = 4(h – 1) + 6</td>
<td>
Each hexagon has six sides, but all of them don't count. So you have to get rid of the vertical ones. So that would be 2x. But you then have to add two back for the sides on the ends.
P = 6h – 2h + 2</td>
</tr>
</table>

INSTITUTE for LEARNING

Hexagon Pattern Task

Teacher: Patricia Rossman
District: Austin Independent School District
Grade: 6

Student: Twenty-two plus 4 is 26; 26 plus 4 is 30, and 30 plus 4 is 34, 34 plus 4, 38; and 38 plus 4 is 42.

Teacher: Okay. So you're telling me you saw a pattern here in the numbers?

Student: Yeah.

Teacher: Well, how could you find the perimeter of the tenth train if you didn't have this information? Would there be another way to find the perimeter of the train? Like you're telling me that this perimeter is four more *(points to the fourth train)* than this one *(points to the third train)*. What's another way to find the perimeter if you don't know this?

Student: The – we can start with one, and we know that's six, and then we put a two in *[Inaudible]* and then we think that kinda we can get it. (Student pointing to hexagon.)

Teacher: Why do you think it is that you add four from the picture?

Student: Because right here, we count six, and then we count like this, all the way, and then we – he said that count by four, and you get all the answers.

Teacher: I'm wondering where this thing that you're talking about, the four all the time, where is the four in the picture?

Student: Right here. One, two, three –

[Crosstalk]

Teacher: Like this is – this is *(points to the third train)* four more than this one *(points to the second train)*, right?

Student: Yes.

Teacher: But where in the picture is it four more than this one?

Student: In the middle?

Teacher: What do you mean in the middle? What do you see?

Student: Oh, yeah, because right here, when that is –

Student: Right here is five *(points to the hexagon at the beginning and at the end)*, and right here is four *(points to sides in the middle)*.

Student: Five, and then five, four, four, five… *(points to the sides of a hexagon)*

Student: Because we have to put in another one right here, and this one has got to be one, two, three, four.

Teacher: Ah. What do you think?

Student: Yeah. He's right.

Teacher: What does he mean, where's the four in the picture?

Student: That because if we –

Teacher: How much is on this one, on the end?

Students: Five.

Teacher: How much on this one?

Students: Five.

Teacher: How much here?

Students: Four.

Students: Four.

Teacher: So how could you think about that for the tenth one?

Student: The –

Teacher: Can you imagine in your mind what it looks like?

Student: Yes.

Student: Yeah. No, no, no, no.

Student: The first and the last one is going to be –

Students: Five.

[Crosstalk]

Student: And the other one is going to be four.

[Crosstalk]

Student: In the middle.

Teacher: You should write about that, because that's what it says to do here. Without building the tenth train, write about how you find that perimeter. Can you write that?

Student: Yeah.

Student: Yes.

Teacher: Good. Go ahead.

Teacher: What did you do?

Student: The two—we did— the first two are going to five because it's just – it's just one because the first, the last one is five because they are just one with the first and the last one, and the other ones has two, and then we're going to be four.

Teacher: Okay. Can you come on up here? I want to – I want to post this on the board, and maybe you can come and point what you're talking about.

Student: I'm talking about those two numbers *(points to the first and last hexagon)*, because those has five, and the other one just has four in each one *(points to the two segments on the top and bottom of a hexagon)*. The number who – the every number who is it.

Teacher: Okay. So where did Daniel say he was getting a five from?

Student: The first one and the last one.

Teacher: Can you come point to the fives? Come on and point to where the fives are. You can stay here for a second, Daniel.

Student: Here and here *(points to the first and last hexagon).*

Teacher: Okay. Show me where the five is in the first one. Can you – One, two, three, four, five. Okay. And show me where the five is in the last one.

Student: One, two, three, four, five. *(points to the sides of the hexagon)*

Teacher: Okay, show me where the five is in the last one?

Student: *(Student points to the five sides.)* One, two, three, four, five…

Teacher: Do you understand what he's saying?

Students: Yes.

Teacher: Which train is this?

[Crosstalk]

Student: Ten.

Student: He said that here in the middle, on the hexagon that has four sides, and you know, the side *(points to the sides of the hexagon)*, and the – yeah, right here and right here *(points to the hexagon at the beginning and end of the train)*, that is the hexagon that has five sides.

Teacher: Aha. So how many of the hexagons have five side lengths?

Student: Five. Two.

Teacher: Two of them. And then how many of the hexagons have the four?

[Crosstalk]

Student: Eight.

Students: Eight.

Student: Because there is ten hexagons.

Teacher: Okay. So does Miguel have your idea pretty good?

Students: Yes.

[End of Audio]